Paradoxical phenomena in crowd motion

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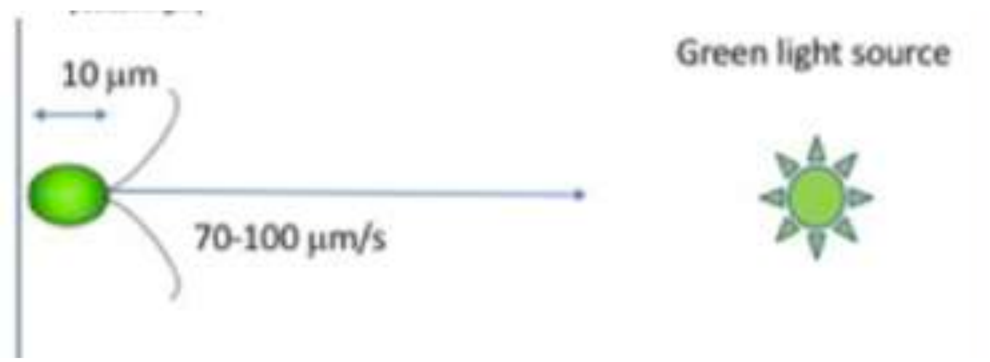
Cargese may 28th 2024

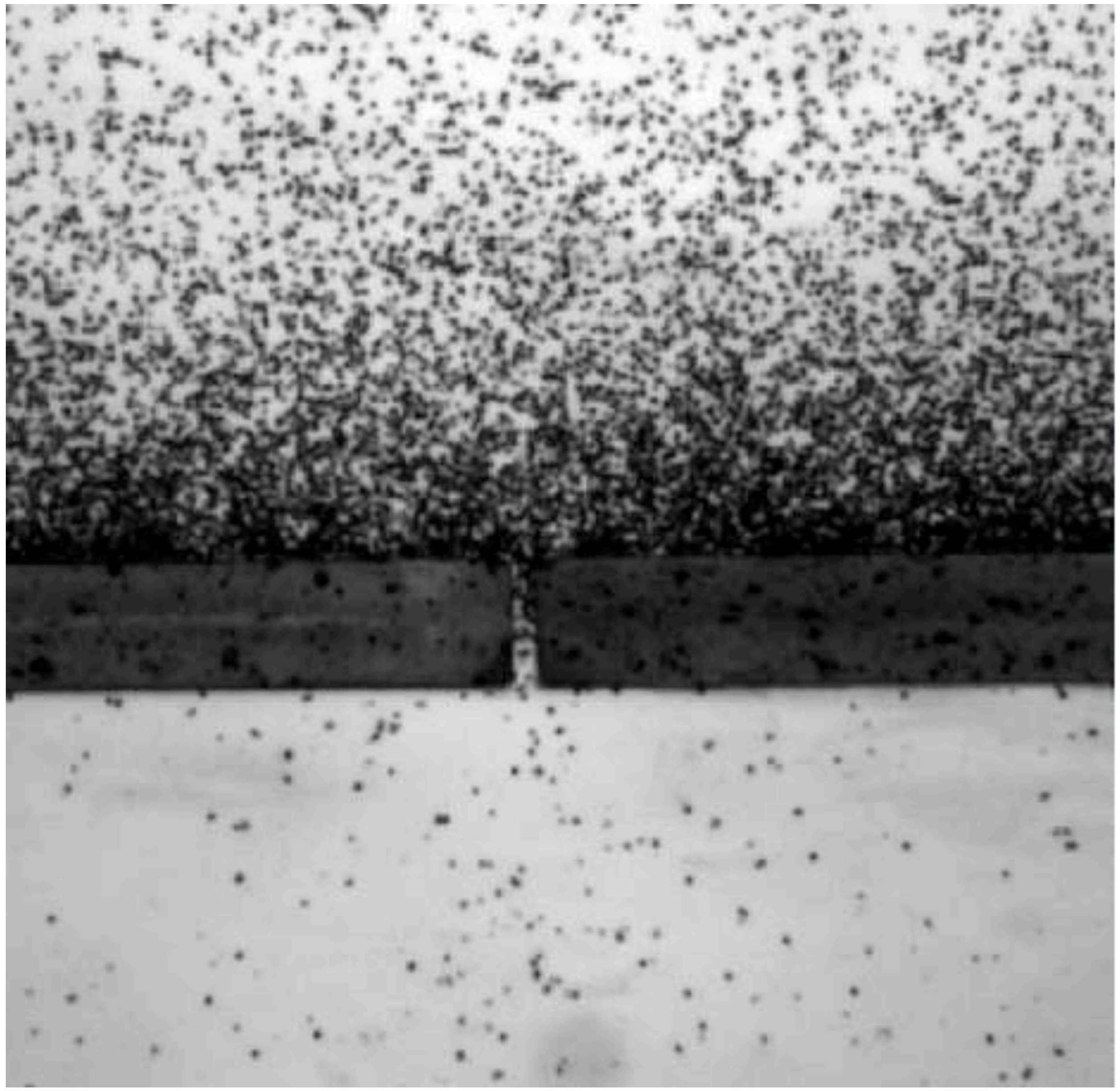
The Conference on Pedestrian and Evacuation Dynamics 2014 (PED2014)

Experimental evidence of the "Faster Is Slower" effect

A. Garcimartín [a,*], I. Zuriguel [a], J.M. Pastor [a], C. Martín-Gómez [b], D.R. Parisi [c]

Chlamydomonas Reinhardtii

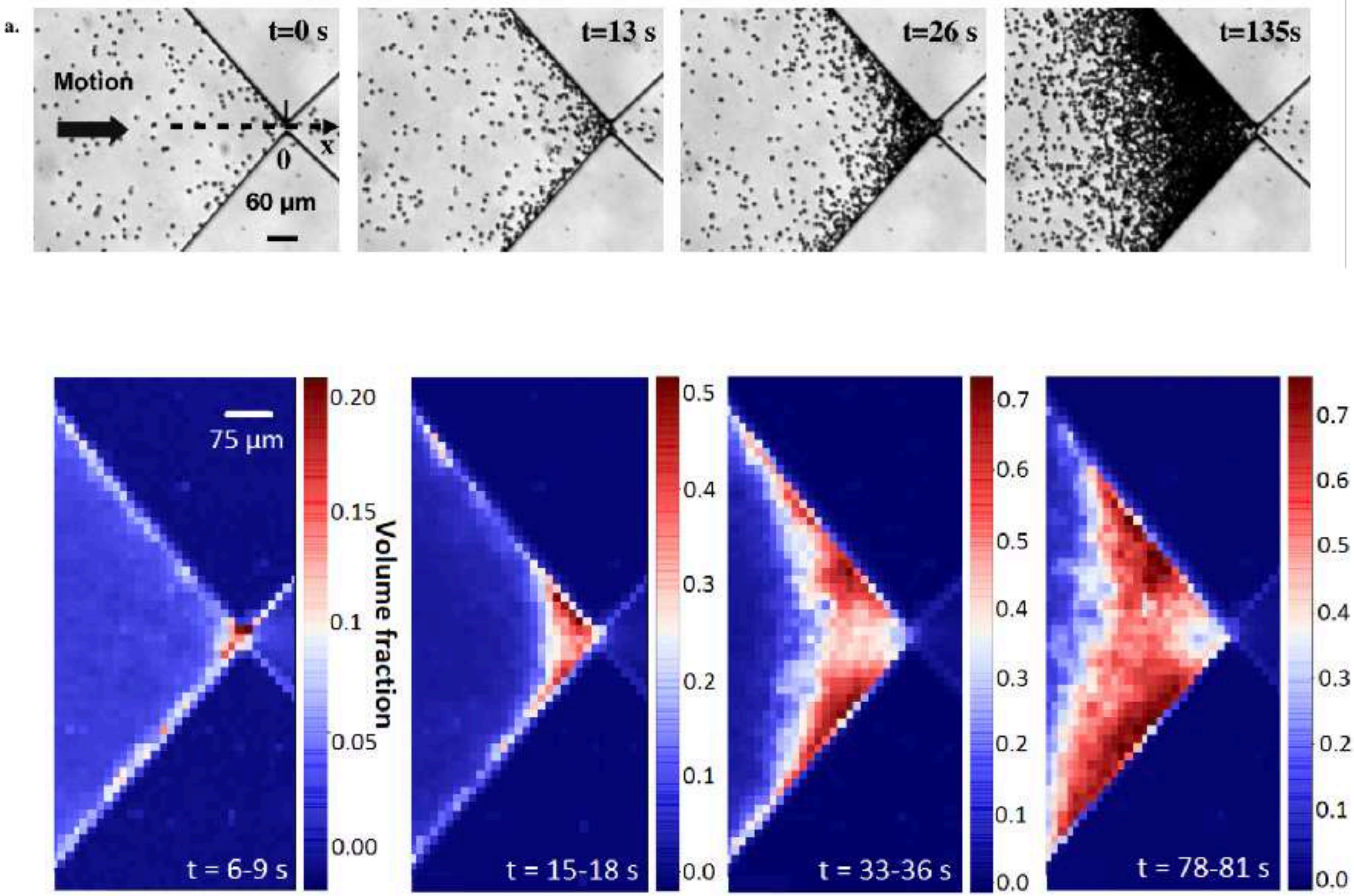


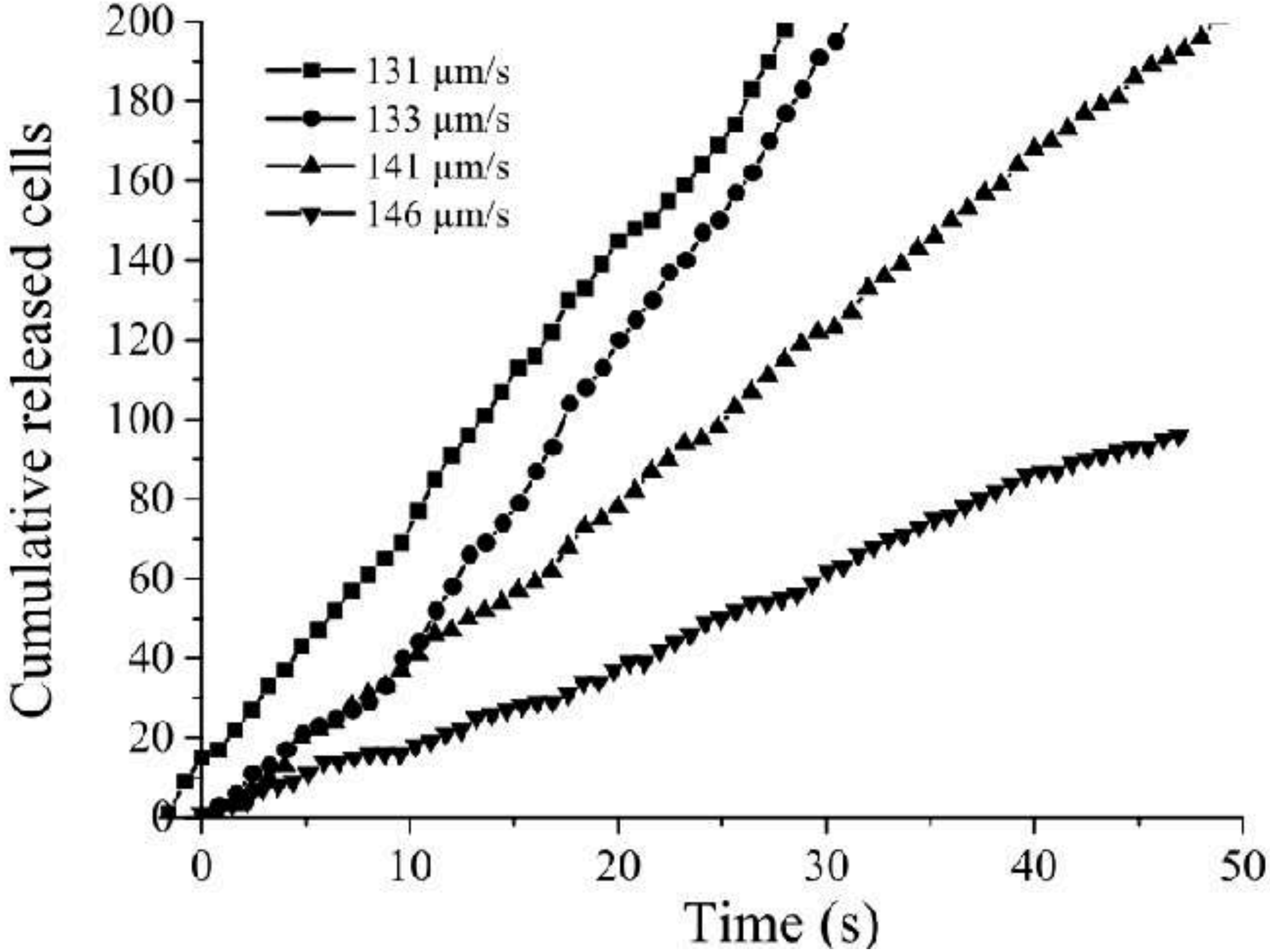


Active jamming of microswimmers at a bottleneck constriction

Edouardo Al Alam, Marvin Brun-Cosme-Bruny, Vincent Borne, Sylvain Faure,
Bertrand Maury, Philippe Peyla, Salima Rafaï

PHYSICAL REVIEW FLUIDS **7**, L092301 (2022)


a.
t=0 s
t=13 s
t=26 s
t=135s
Motion
x
0
60 µm
75 µm
0.20
0.15
0.1
0.05
0.00
Volume fraction
t = 6-9 s
0.5
0.4
0.3
0.2
0.1
0.0
t = 15-18 s
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
t = 33-36 s
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
t = 78-81 s



S. Rafai & P. Peyla (Liphy)

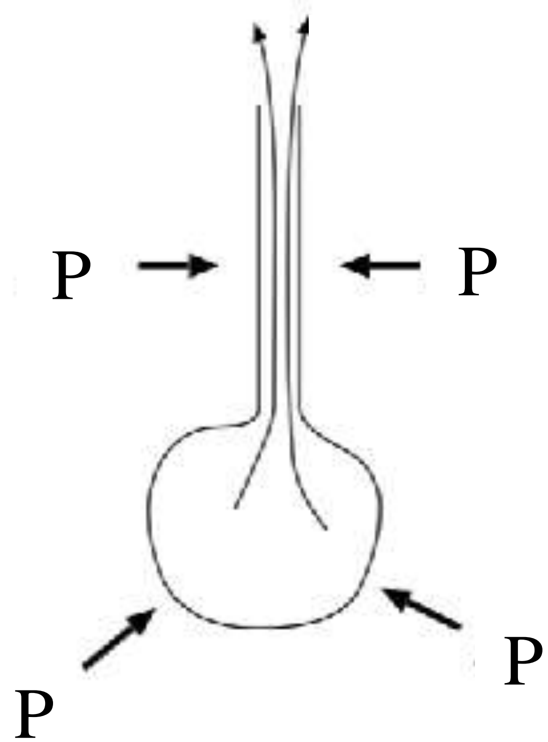
Faster is Slower Effect in natural phenomena

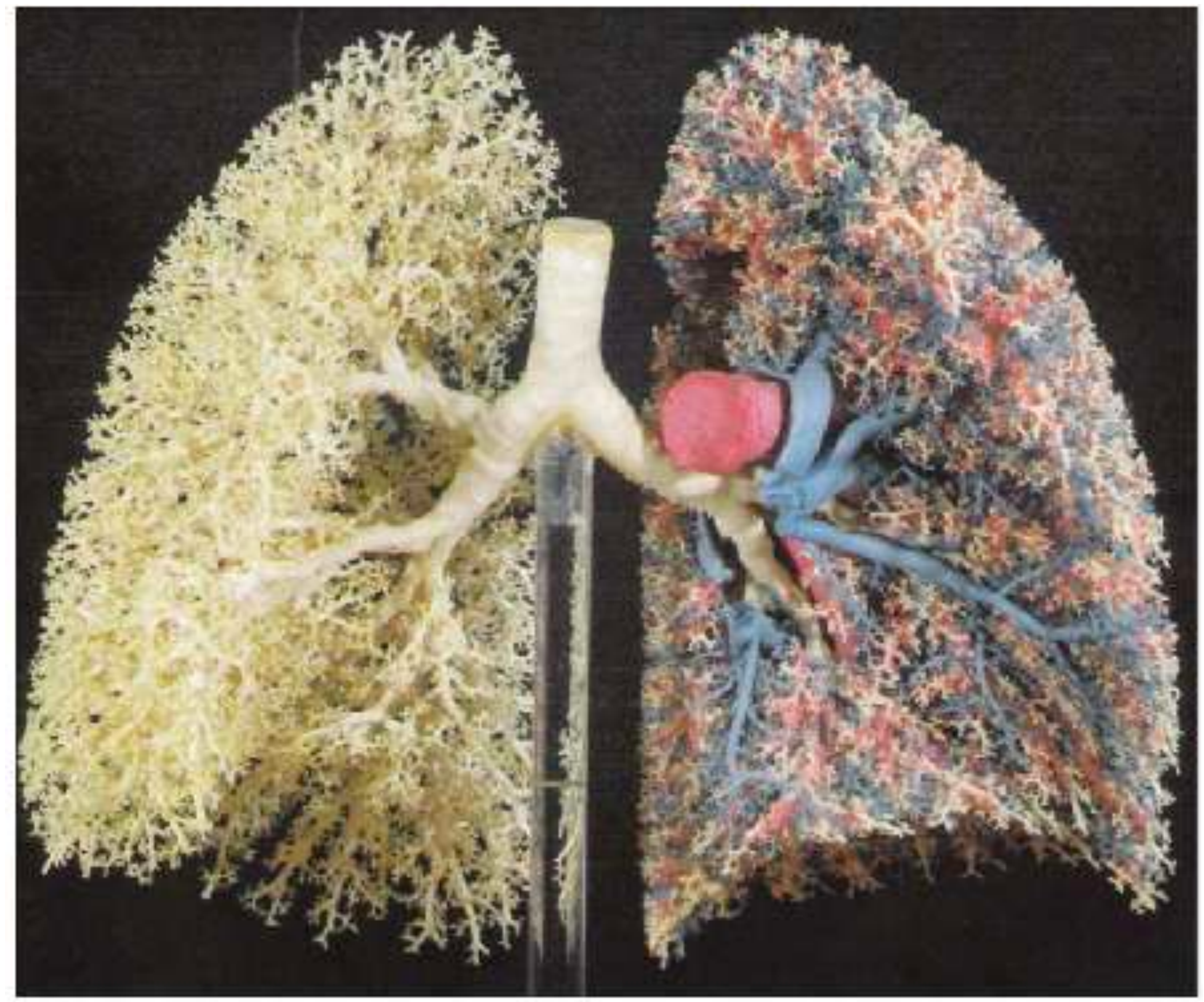
What is better is the enemy of what is good.

For general systems : counter-effective increase of the forcing term
(possibly above a certain threshold)

Examples in other contexts

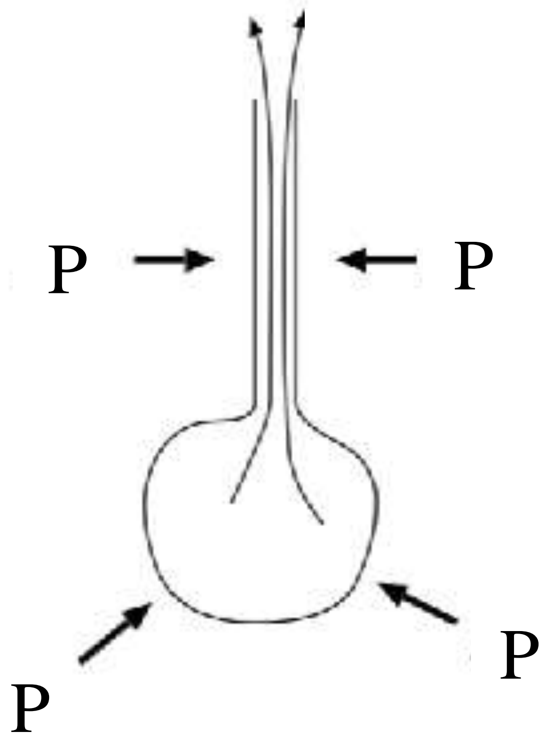
Maximal expiratory flow





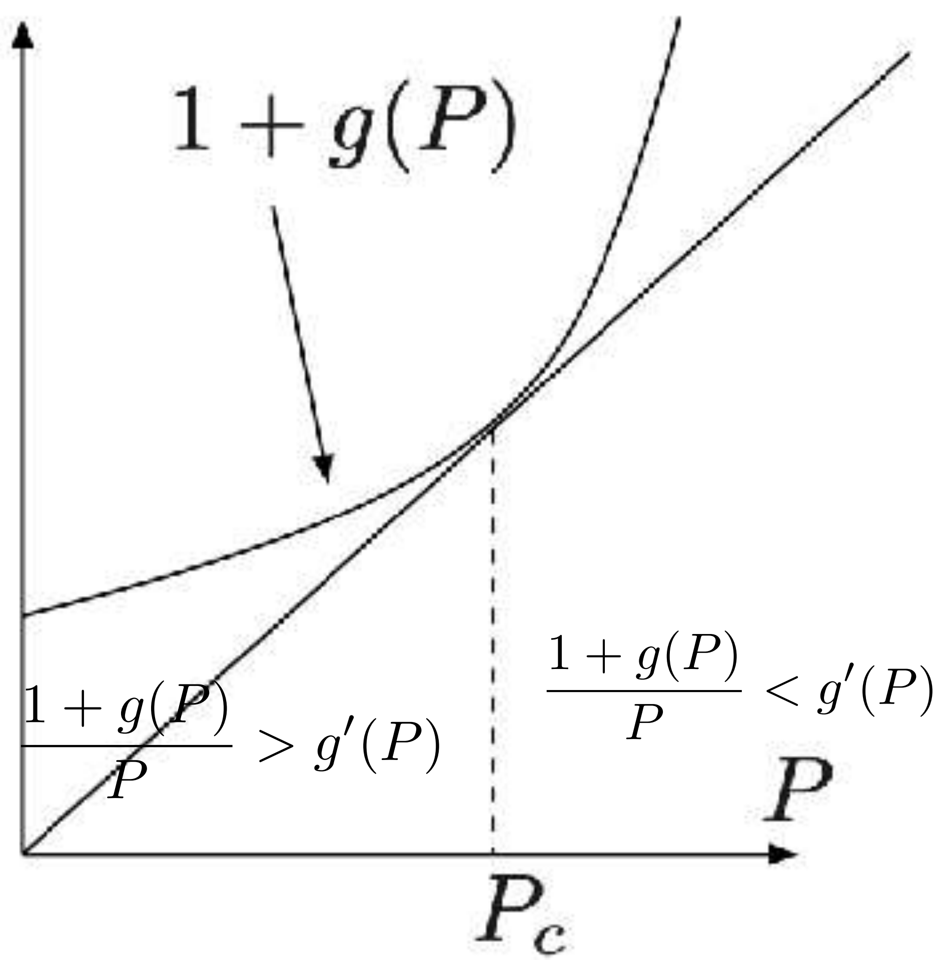
The outflow is *induced* by application of a positive pressure.
This extra pressure in the parenchyma tends to reduce the diameter of the branches, thus *harming, and possibly stopping*, the expiration process

Faster is Slower Effect in natural phenomena



$$R(P) = R_0(1 + g(P)) \qquad g(0) = 0.$$

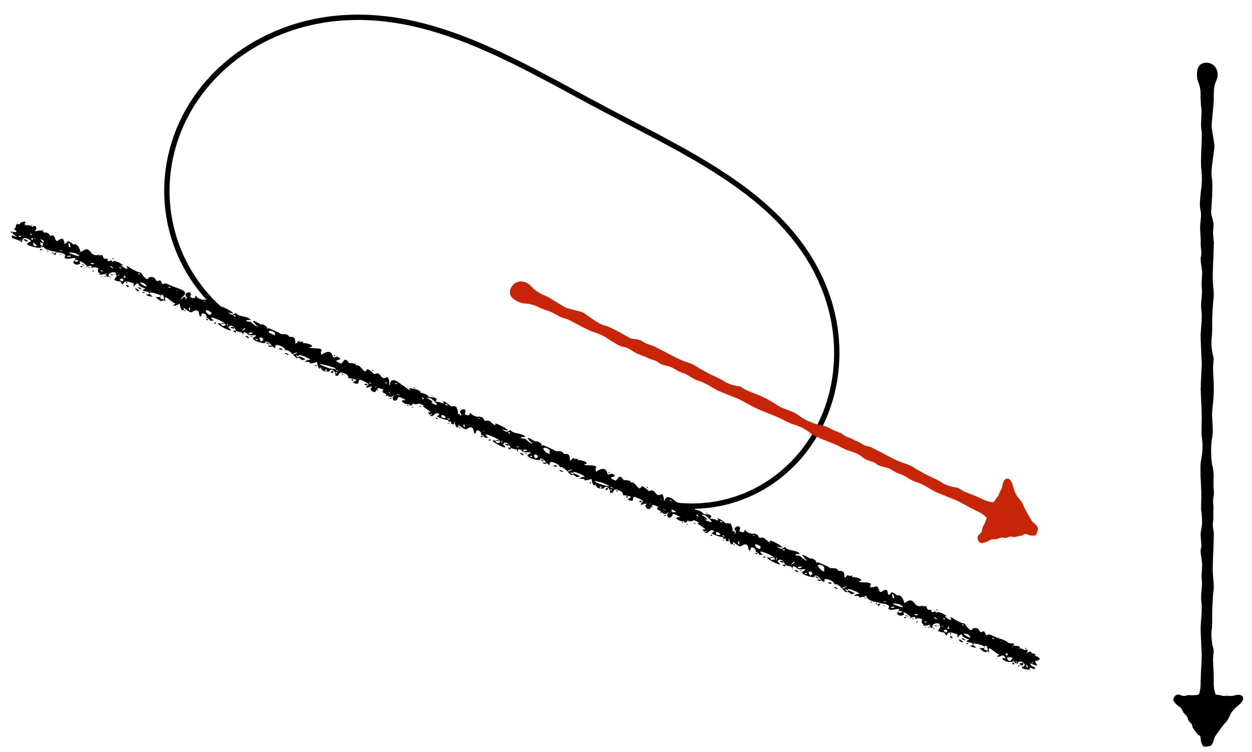
$$Q = \frac{P}{R} = \frac{P}{R_0(1+g(P))} \Longrightarrow \frac{dQ}{dP} = \frac{1}{R_0}\frac{1+g(P) - Pg'(P)}{(1+g(P))^2}$$



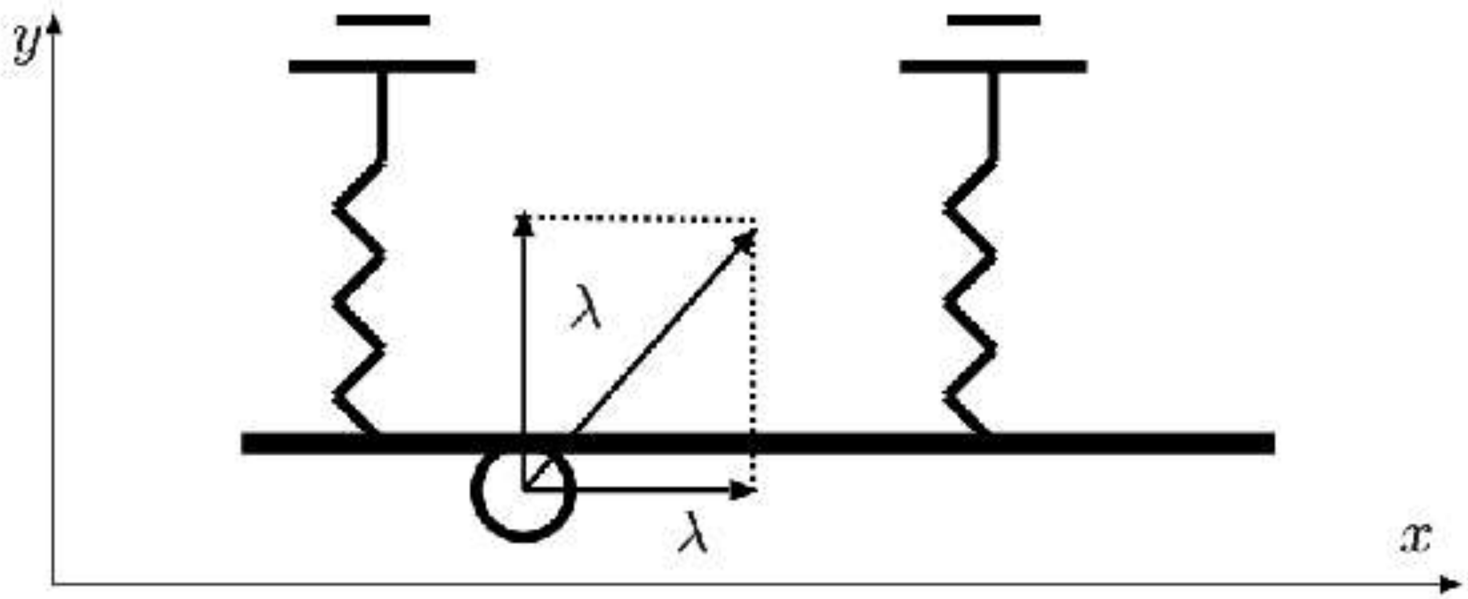
Faster is Slower Effect

Hurtling Droplet

de Gennes, P., Brochard-Wyart, F., 2015. Gouttes, bulles, perles et ondes. Échelles, Humensis. URL: https://books.google.fr/books?id=DhSWDgAAQBAJ.

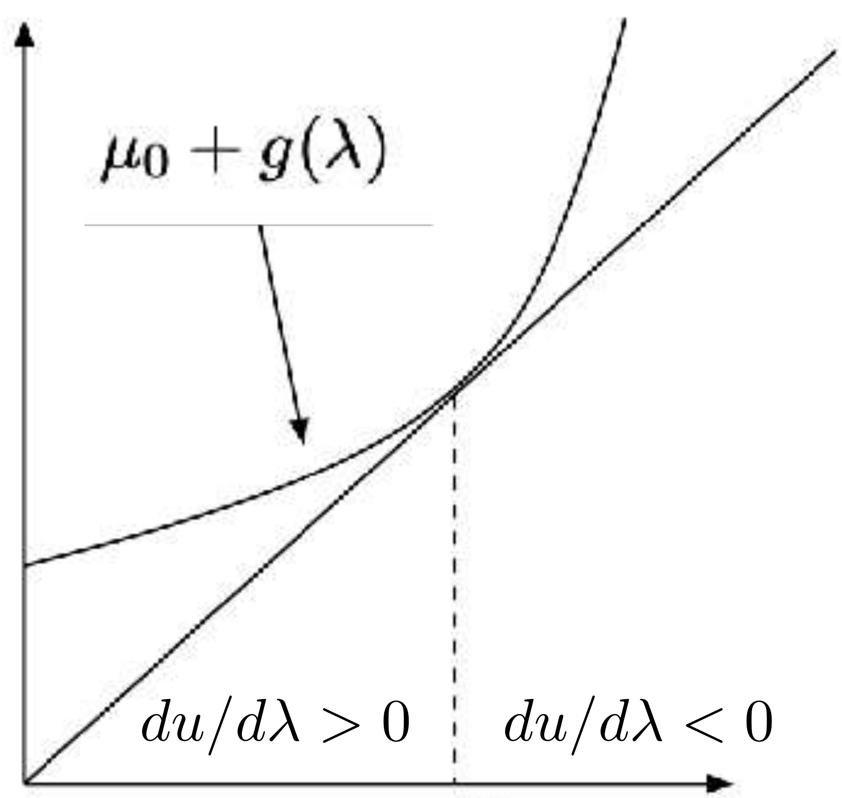


A simple frictional system



Viscous force $-\mu(\lambda)u$

$$\mu(\lambda) = \mu_0 + g(\lambda)$$



Horizontal velocity $u = \dfrac{\lambda}{\mu(\lambda)}$

$$du/d\lambda = \frac{\mu(\lambda) - \lambda\mu'(\lambda)}{\mu(\lambda)}$$

Faster is Slower Effect in crowd motion modeling

Remark : why is it not so trivial to reproduce ?
Consider a general class of models :

$$\frac{dq}{dt} = F_{U^\star}(q(t))$$

Where U^* (parameter) quantifies the eagerness of people to exit a room

Assume $U^\star \longmapsto F_{U^\star}(q)$ is positively 1- homogeneous

Faster $U^\star \longmapsto \lambda U^\star, \ \lambda > 1$

$q_\lambda(t) = q(\lambda t)$ is a solution (The « movie » is accelerated)

$\longrightarrow$ Faster is faster effect

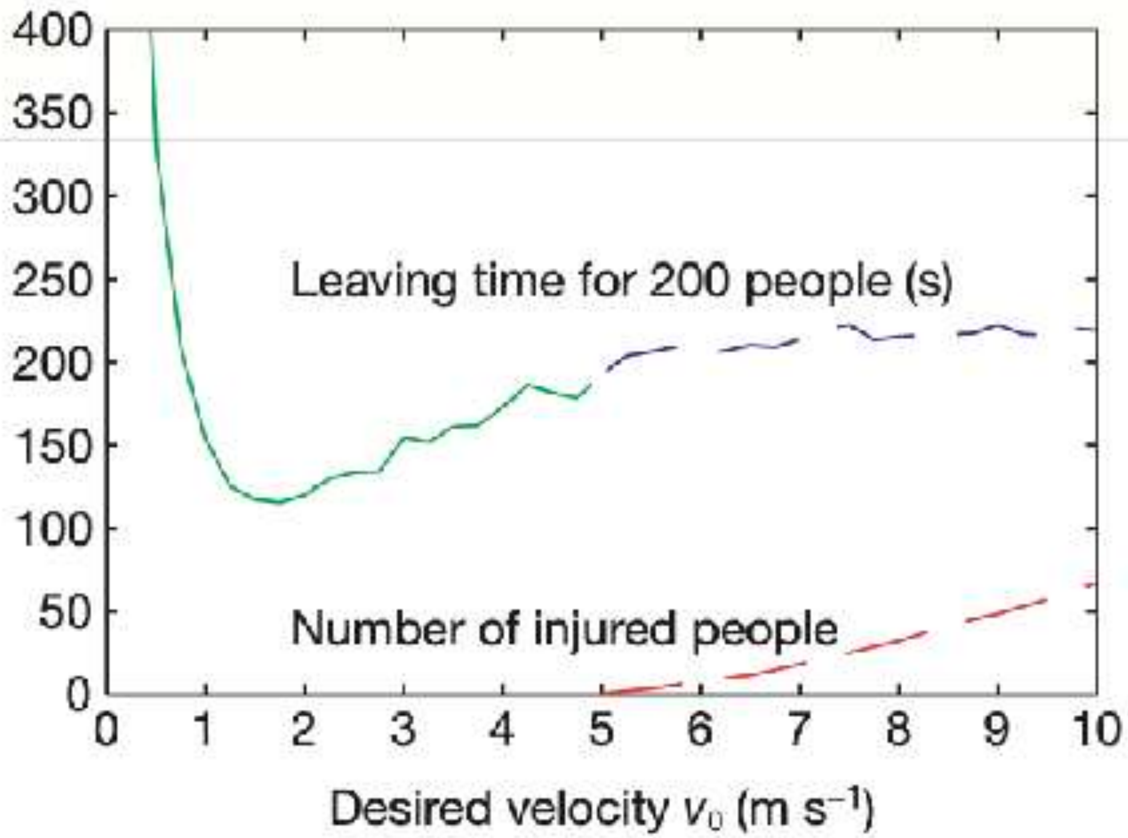
Faster is Slower Effect in crowd motion modeling

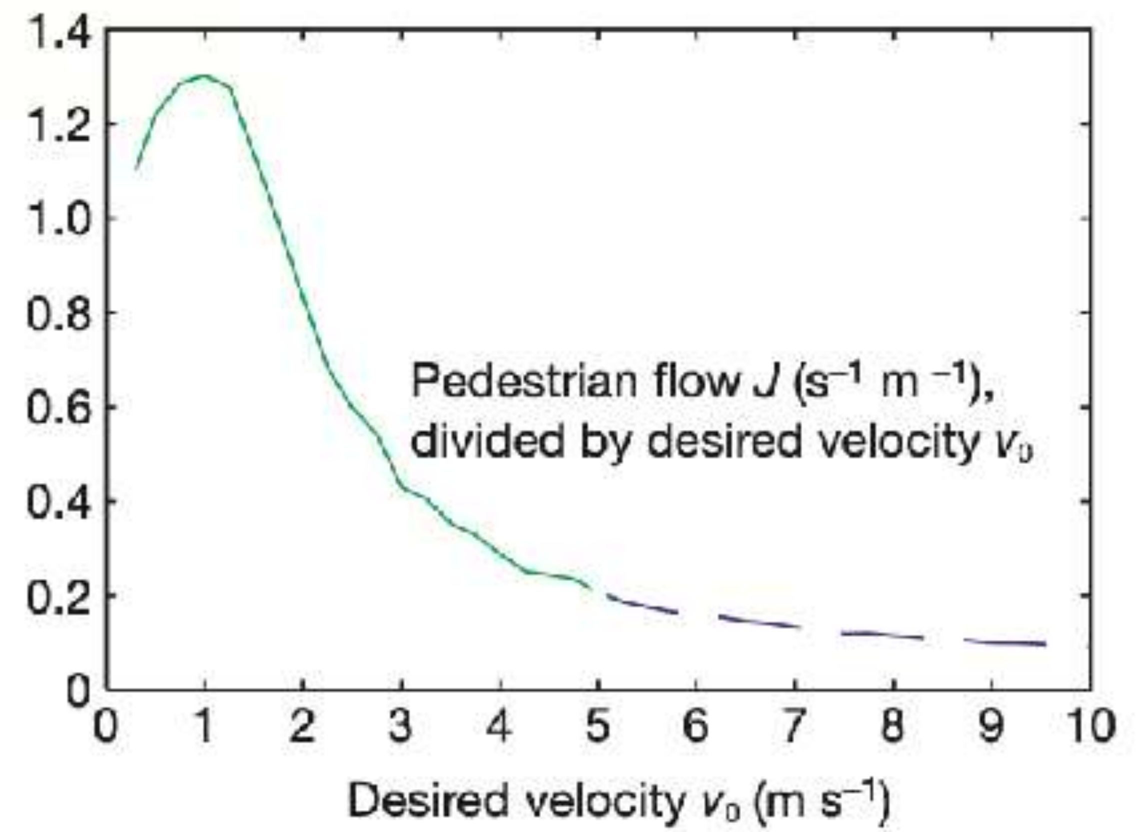
Social force model :

$$m_i \frac{du_i}{dt} = \frac{m_i}{\tau}(U_i - u_i) + \sum_{j \neq i} f_{ij} + \sum_k f_{ik}^w$$

Reproduces the FiS effect with an additional friction term

$$f_{ij}^{friction} = \eta \left(D_{ij}\right)_- \left((u_i - u_j) \cdot e_{ij}^{\perp}\right) e_{ij}^{\perp}$$

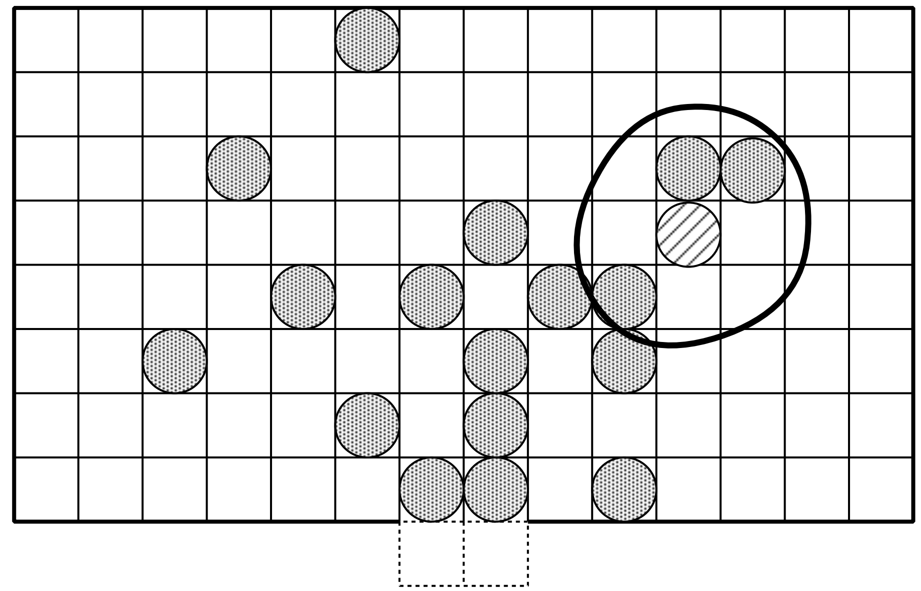


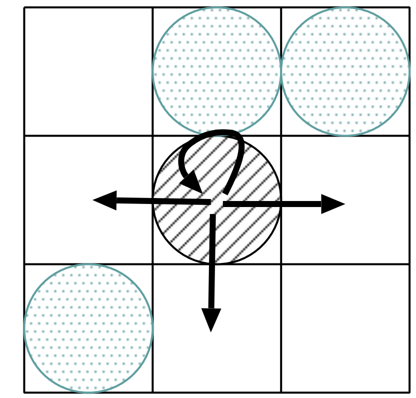


Helbing, D., Farkas, I., Vicsek, T., 2000. Simulating dynamical features of escape panic. Nature 407, 487–490.

Faster is Slower Effect in crowd motion modeling

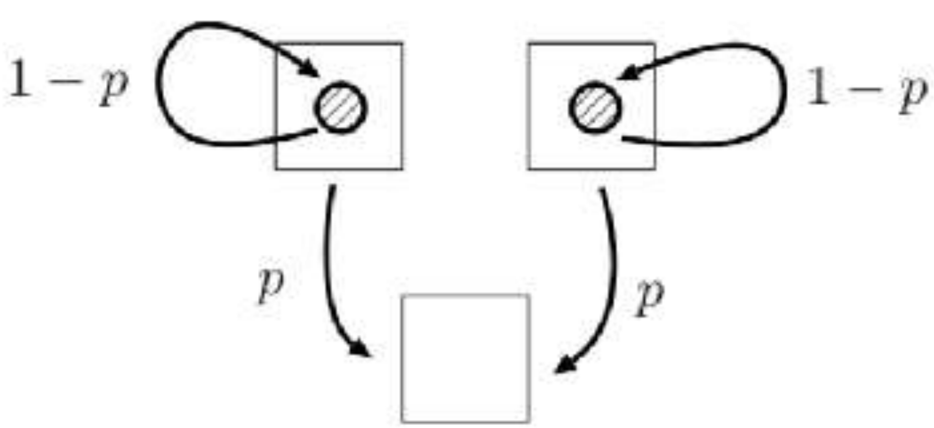
Cellular Automata

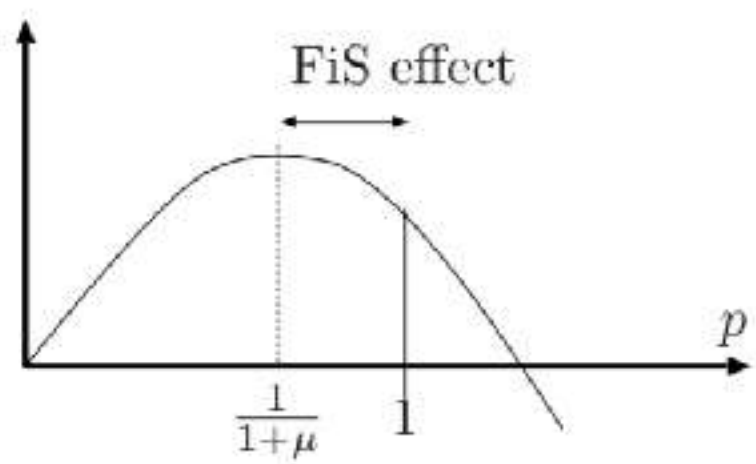




Again, friction makes it work: in case of a conflict, there is a non-zero probability that no competitor moves

Kirchner, A., Klüpfel, H., Nishinari, K., Schadschneider, A., Schreckenberg, M., Simulation of competitive egress behaviour , *Physica A*, 324: 689-697, 2002.



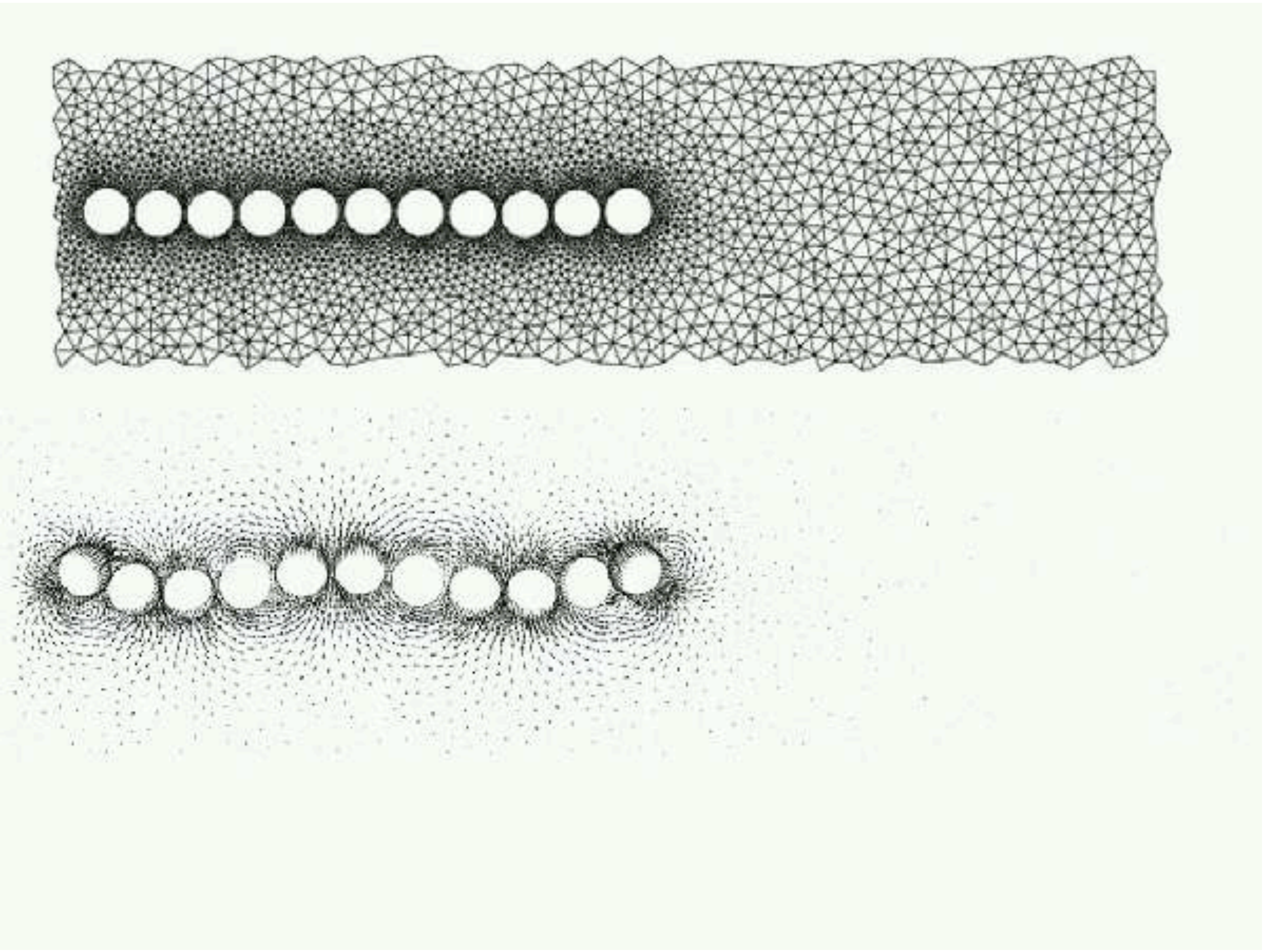


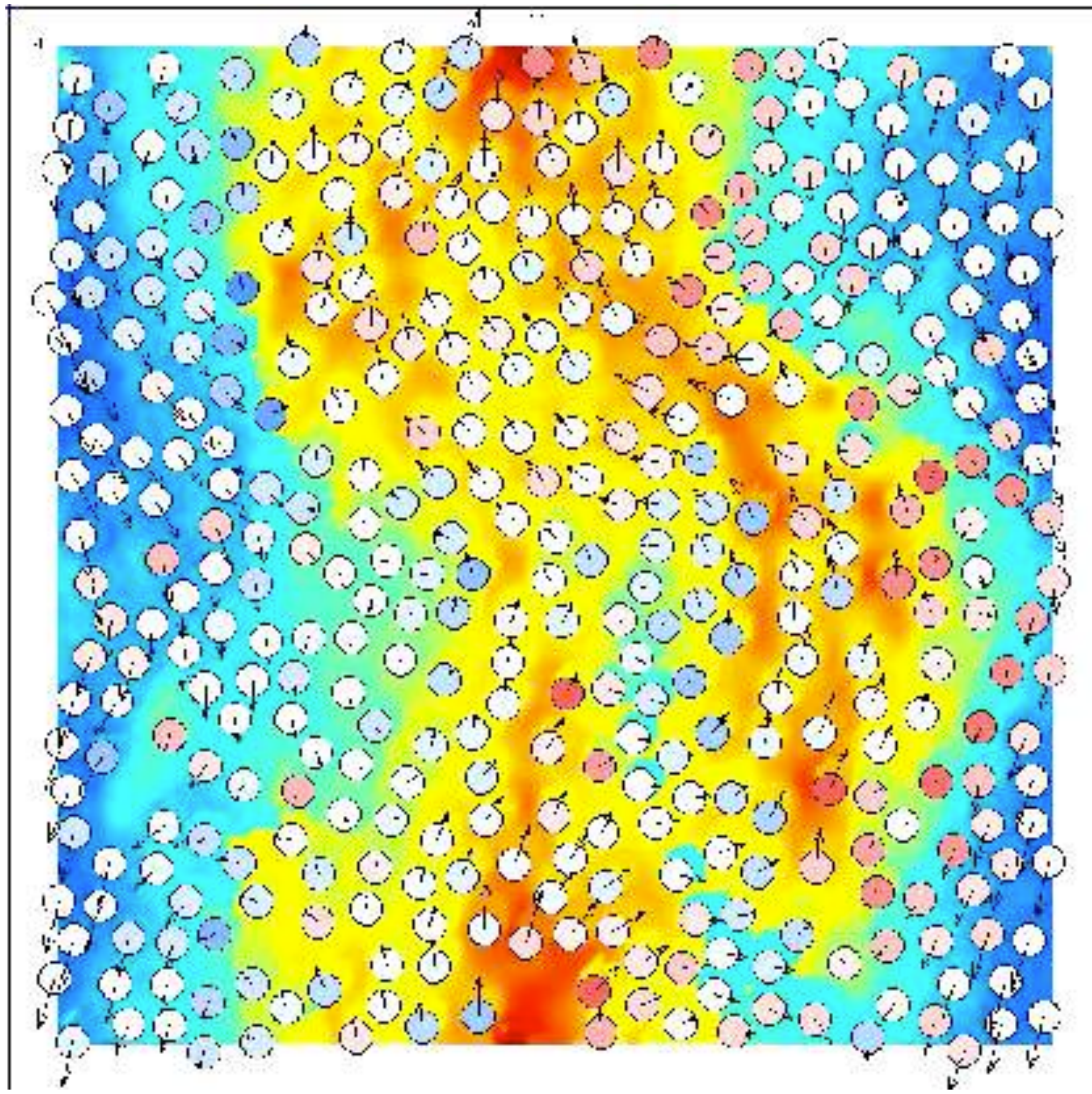
Faster is Slower Effect

Alternative standpoint, an underlying *bizarre* Laplace operator

A granular (purely selfish) model (with J. Venel)

Initially developped to handle collisions in fluid grain simulations



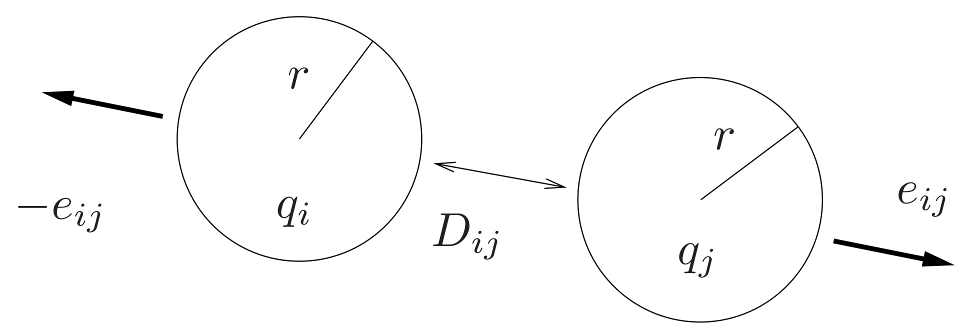


A granular (purely selfish) model (with J. Venel)

N individuals, centered at $q_1, q_2, \ldots, q_N \in \mathbb{R}^2$

Set of feasible configurations $K = \{q \in \mathbb{R}^{2N}, D_{ij}(q) = |q_j - q_i| - 2r \geq 0 \quad \forall i \neq j\}$

Spontaneous velocities $U = (U_1, \ldots, U_N)$



$$u = \frac{dq}{dt} = P_{C_q} U$$

$$C_q = \{v\,,\ D_{ij}(q) = 0 \Rightarrow e_{ij} \cdot (v_j - v_i) \geq 0\}$$

A granular (purely selfish) model (with J. Venel)

$$u = \frac{dq}{dt} = P_{C_q} U$$

$$C_q = \{v\,,\ D_{ij}(q) = 0 \Rightarrow e_{ij} \cdot (v_j - v_i) \geq 0\}$$

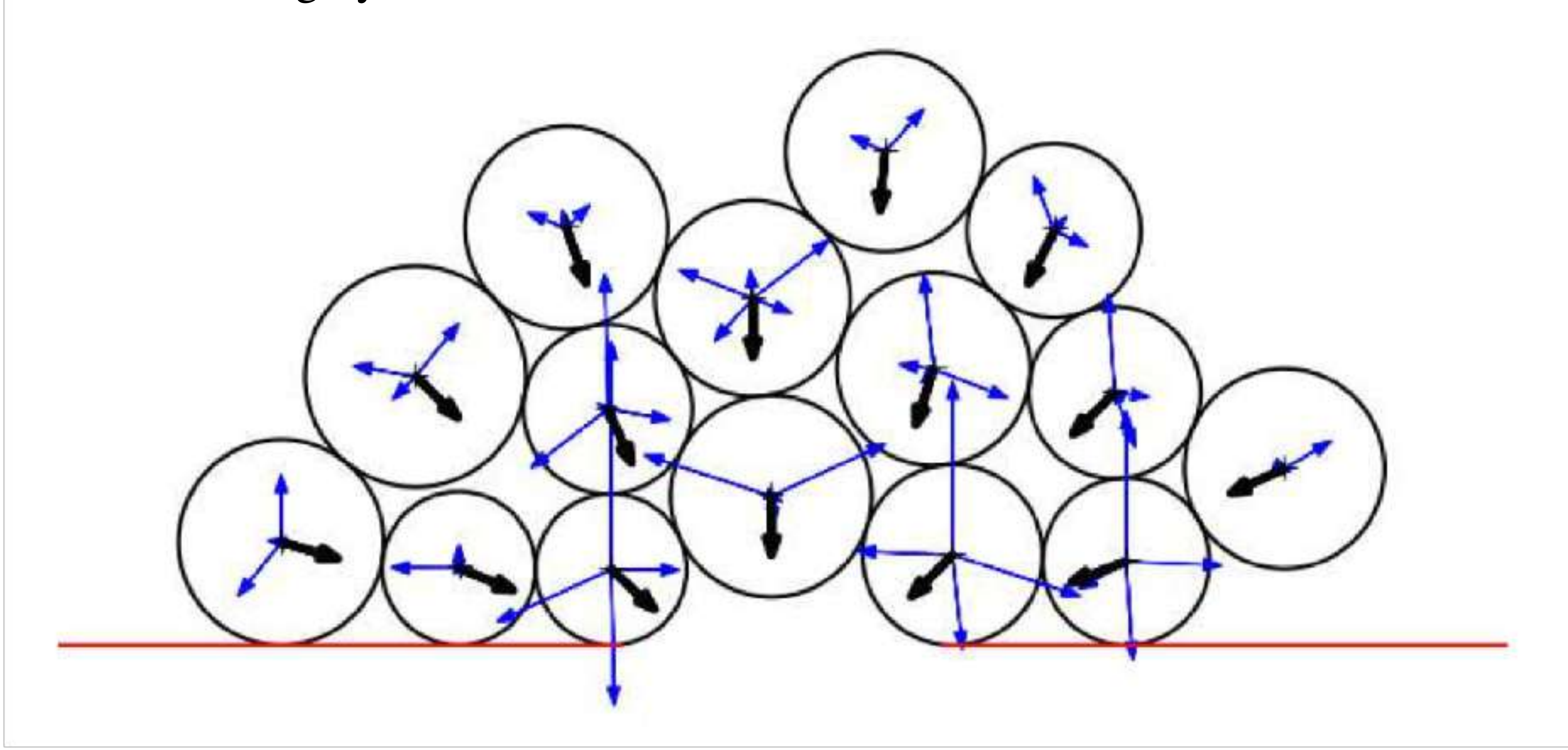
$$\frac{dq}{dt} \in -\partial(\Psi(q) + I_K(q)) \qquad \Psi(q) = \sum_i D(q_i) + I_K(q)$$

$$I_K(q) = \left| \begin{array}{ll} 0 & \text{if } \ q \in K \\ +\infty & \text{if } \ q \notin K \end{array} \right.$$

N.B. The model is *1-positively homogeneous*
No FiS can be reproduced by simply increasing the speed

A granular (purely selfish) model (with J. Venel)

Ambiguous role of « desired velocities », actually closer to individual forces in highly crowded situations



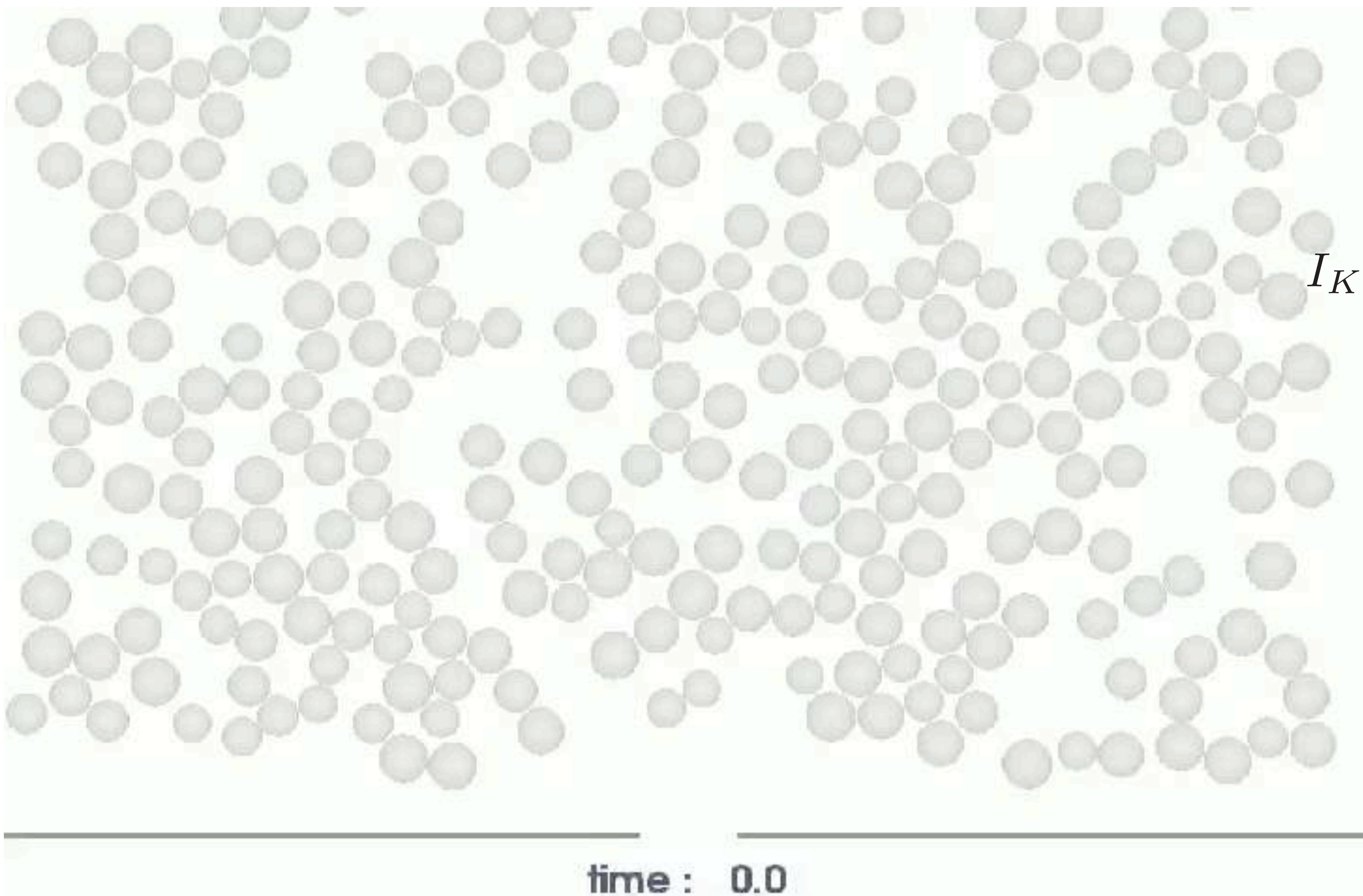
Static jam

Individual dissatisfaction
(distance to the exit)

Gradient flow framework

Dissatisfaction functional $$\Psi(q) = \sum_i D(q_i) + I_K(q)$$

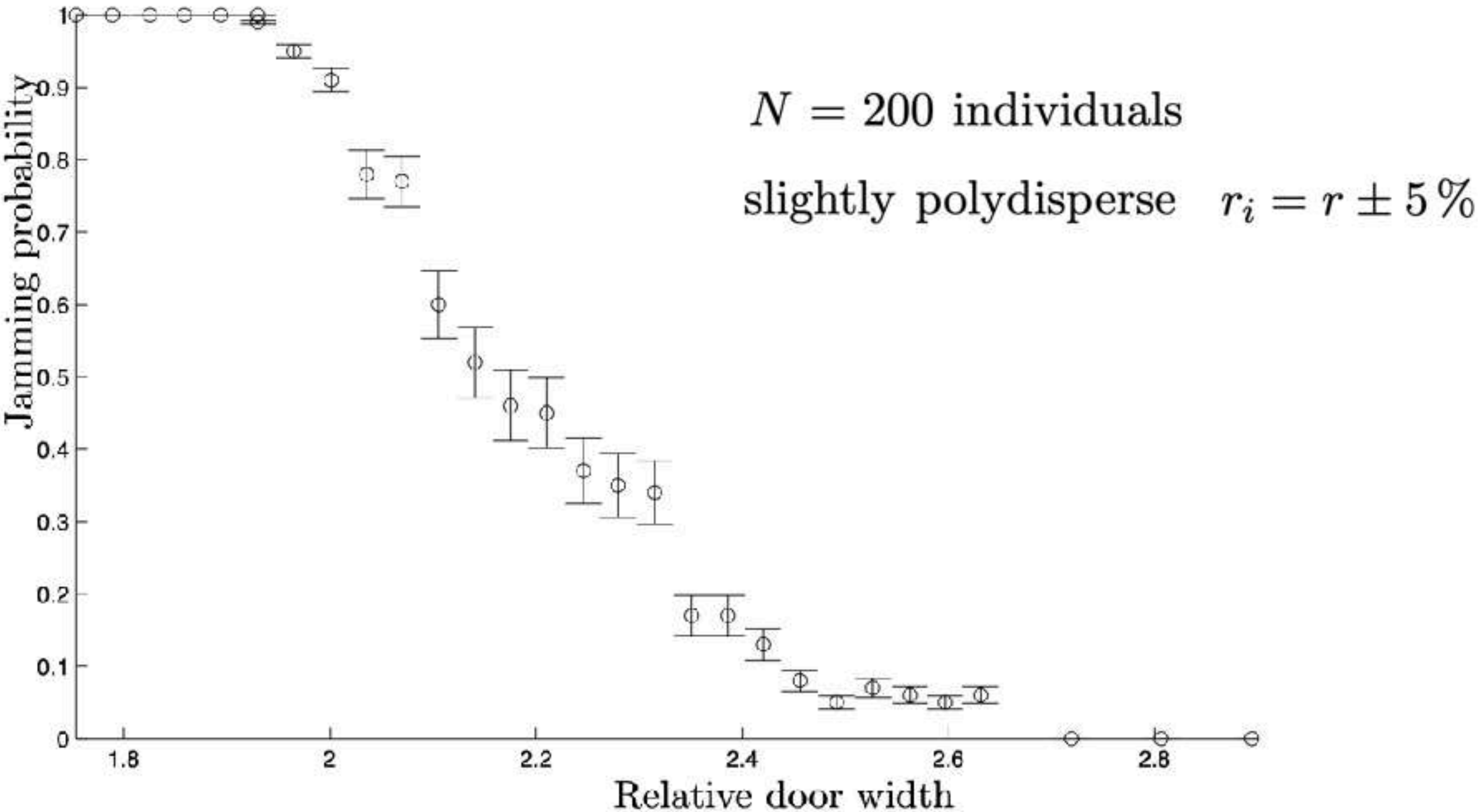
$$\frac{dq}{dt} \in -\partial\Psi(q) \qquad \partial\Psi(q) = \{v,\ \Psi(q) + v \cdot h \leq \Psi(q+h)\ \ \forall h\}$$



Non overlapping
constraint

$$I_K(q) = \left| \begin{array}{ll} 0 & \text{if } \ q \in K \\ +\infty & \text{if } \ q \notin K \end{array} \right.$$

Probability of clogs with respect to door width



Faure, S., Maury, B., 2015. Crowd motion from the granular standpoint. Mathematical Models and Methods in Applied Sciences 25, 463–493.

Estimation of forces within crowds

Horizontal force exerted by an individual :

30 to 75 % of their weight (J. Fruin « Cause and Prevention of Crowd disasters »)

Force de poussée par rapport au poids (N/kg)				
	Sans bras		*Avec bras*	
	Moyenne	Écart type	Moyenne	Écart type
Avants	18,8	1,3	24,5	3,2
Arrières	20,3	3,6	23,4	3
Non-rugbymen	10,7	2,7	13,3	2,4

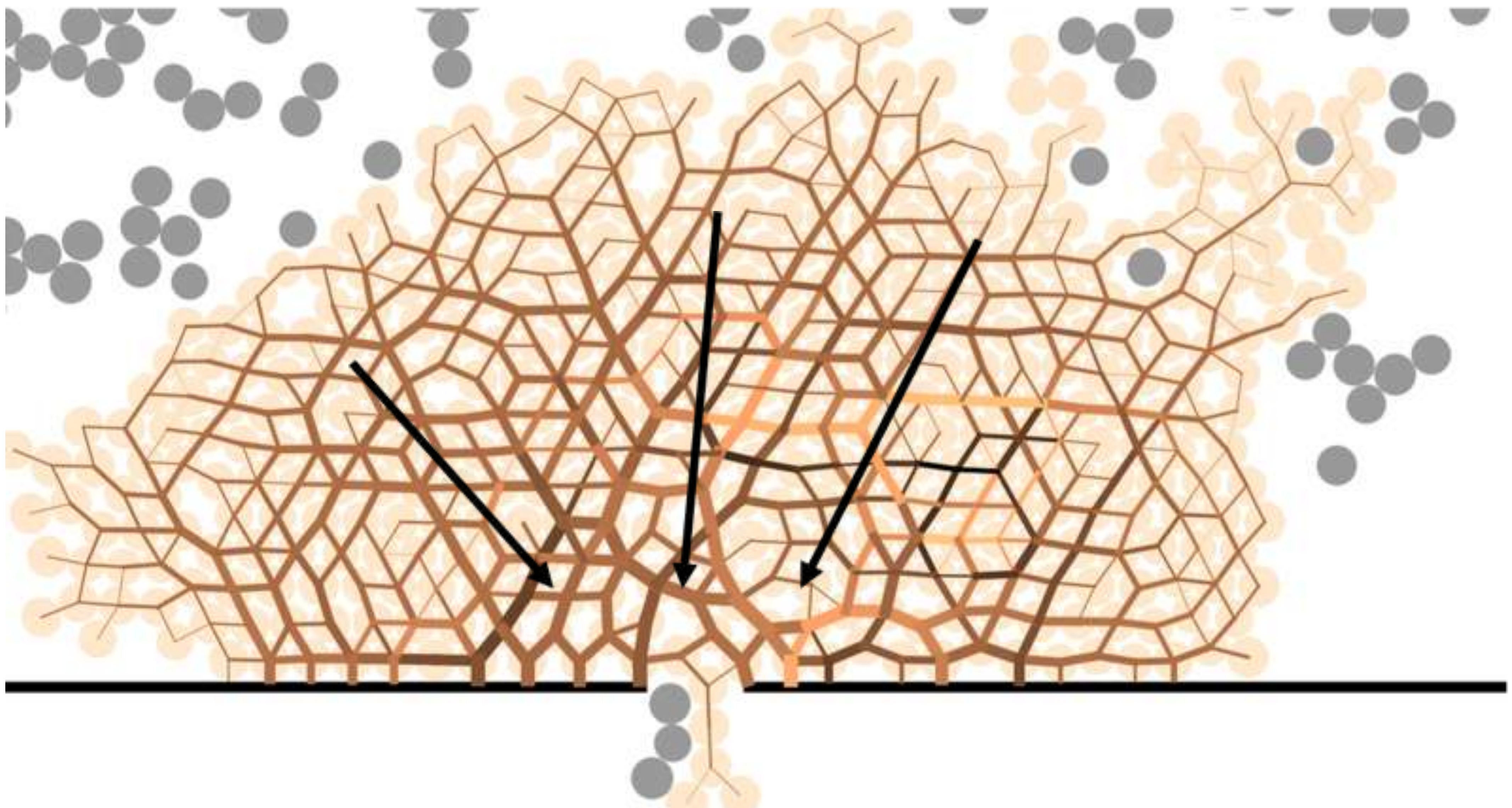
T Ben Khélifa[1], JP Micallef[1], A Quilis[1], JF Brun[2], A Orsetti[2] **Biomécanique de la force de poussée en mêlée de rugby** *Science & Sports* (1995) 10, 163–164 © Elsevier, Paris

In a crowd :

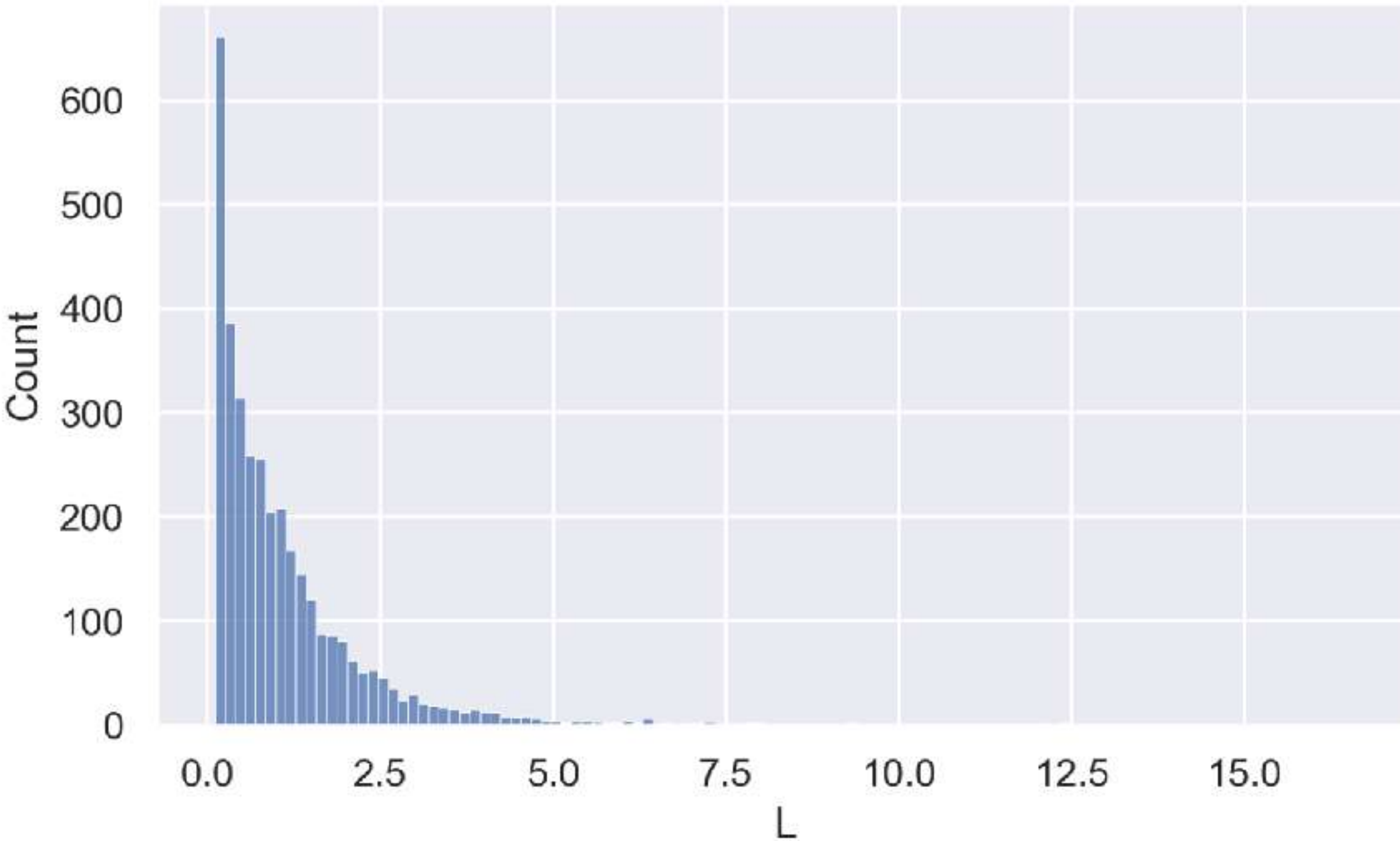
Estimated forces : ~ 450 kg

Thoracic cage crushing ~ 250 kg

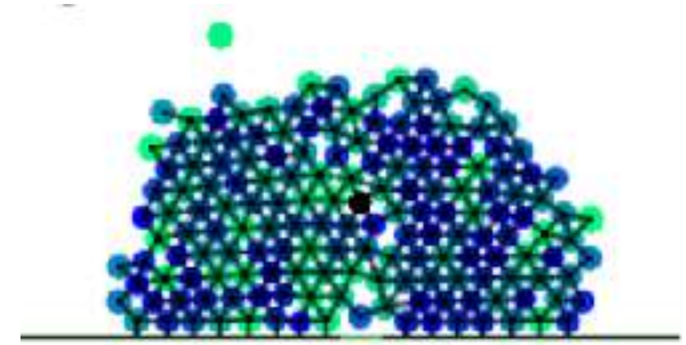
Shaped charge effect

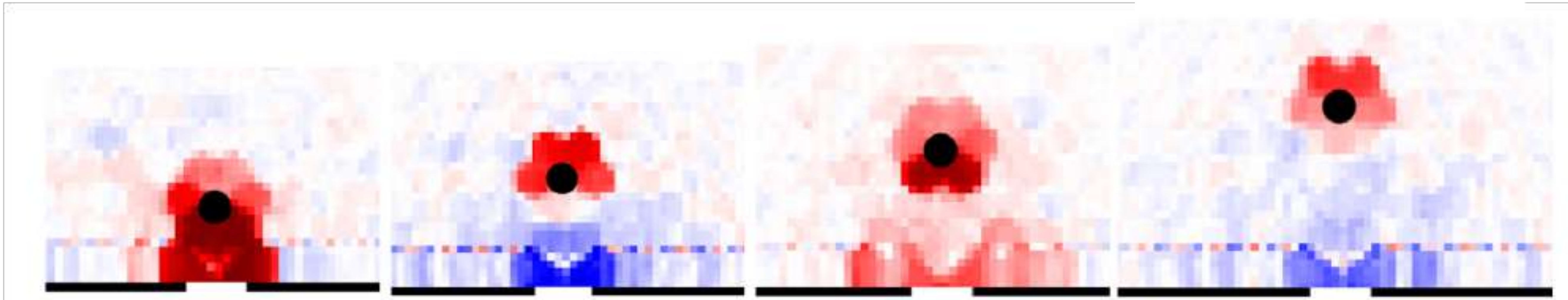


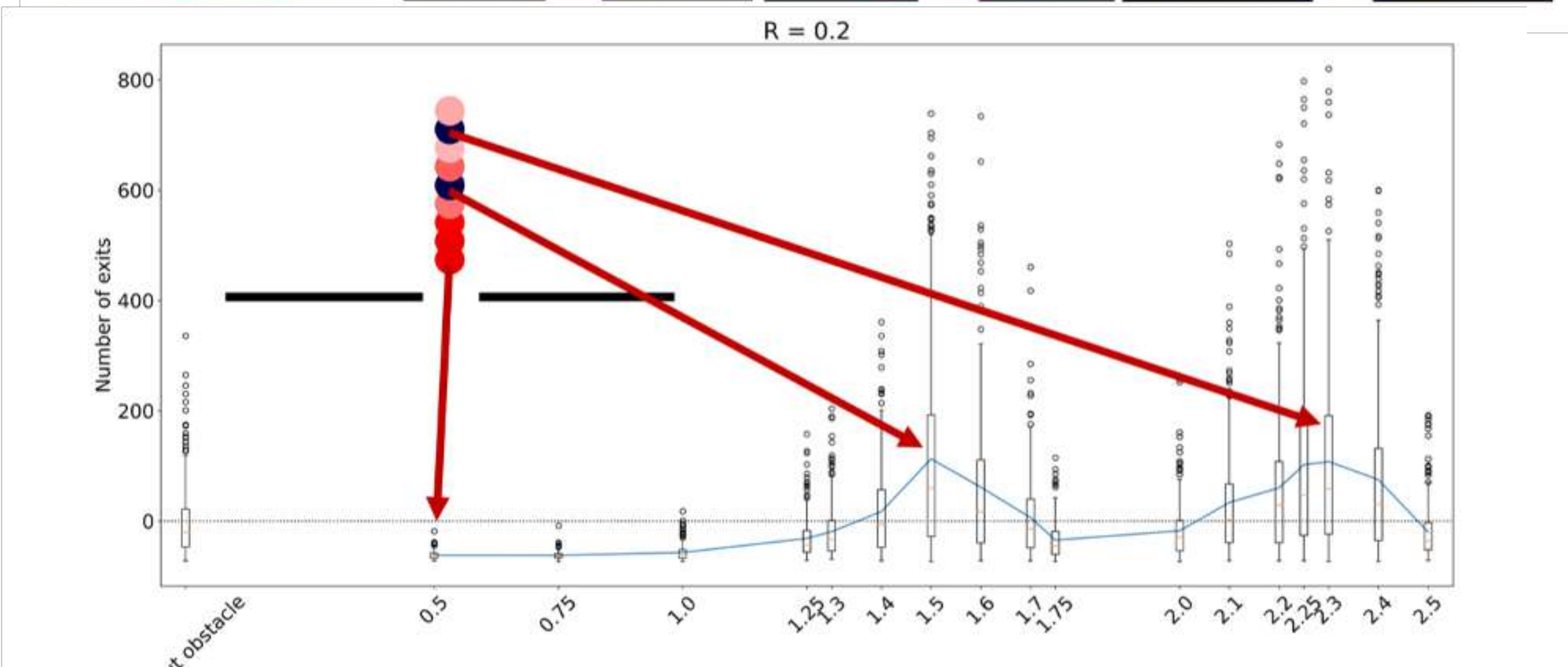
Histogram of « forces » (relatively to individual forces)



Effect of an obstacle upon inter individual forces
Color expresses différence with respect to the no obstacle case
(Higher pressure in red, lower pressure in blue)





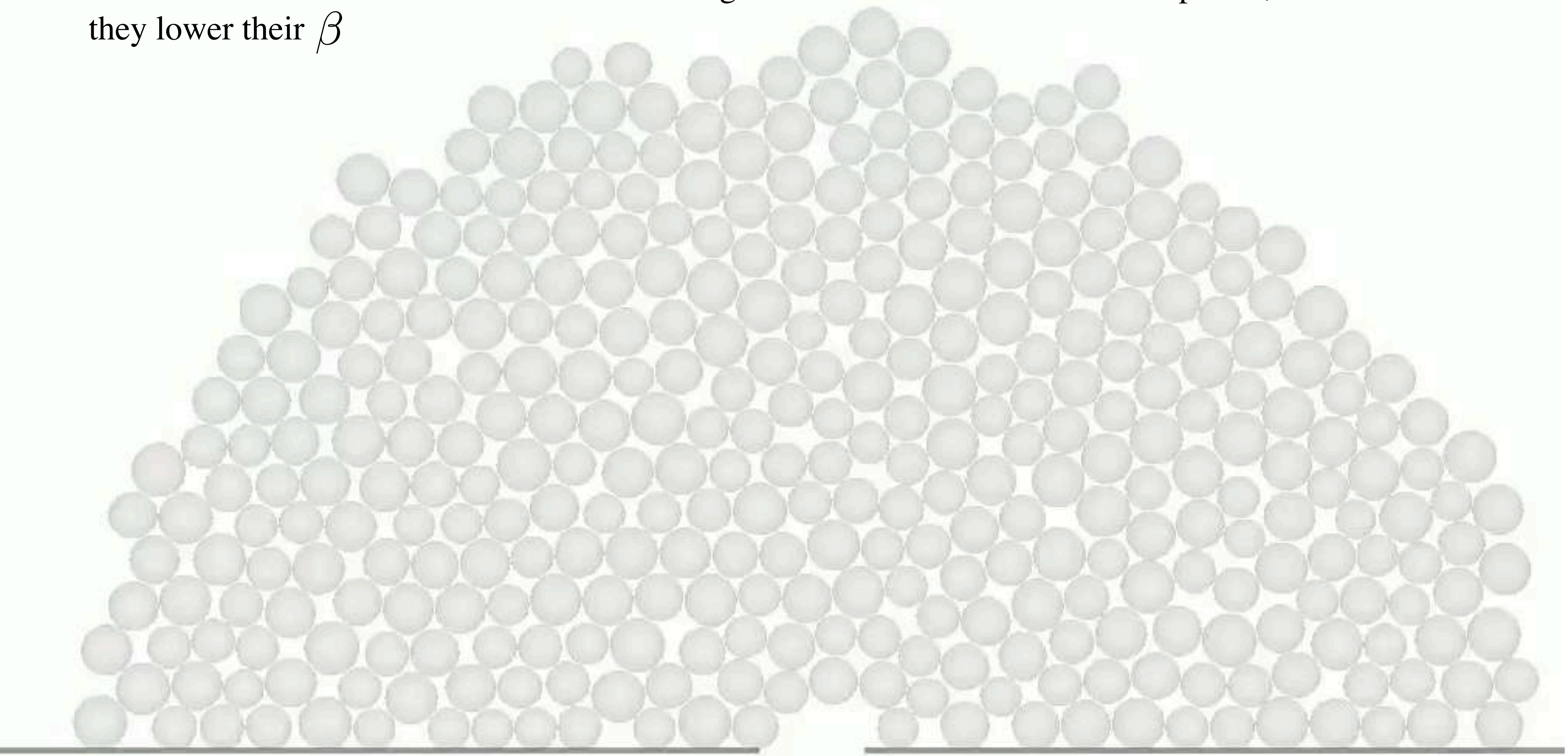


Speeding up evacuation by cooling down the crowd (**Slower is Faster** effect)

$$\Psi(q) = \sum_{i=\mathbf{1}}^{N} \beta_i D(q_i) + I_K(q) \quad \beta_i \in [0,1] \qquad\qquad \frac{dq}{dt} \in -\partial\Psi(q)$$

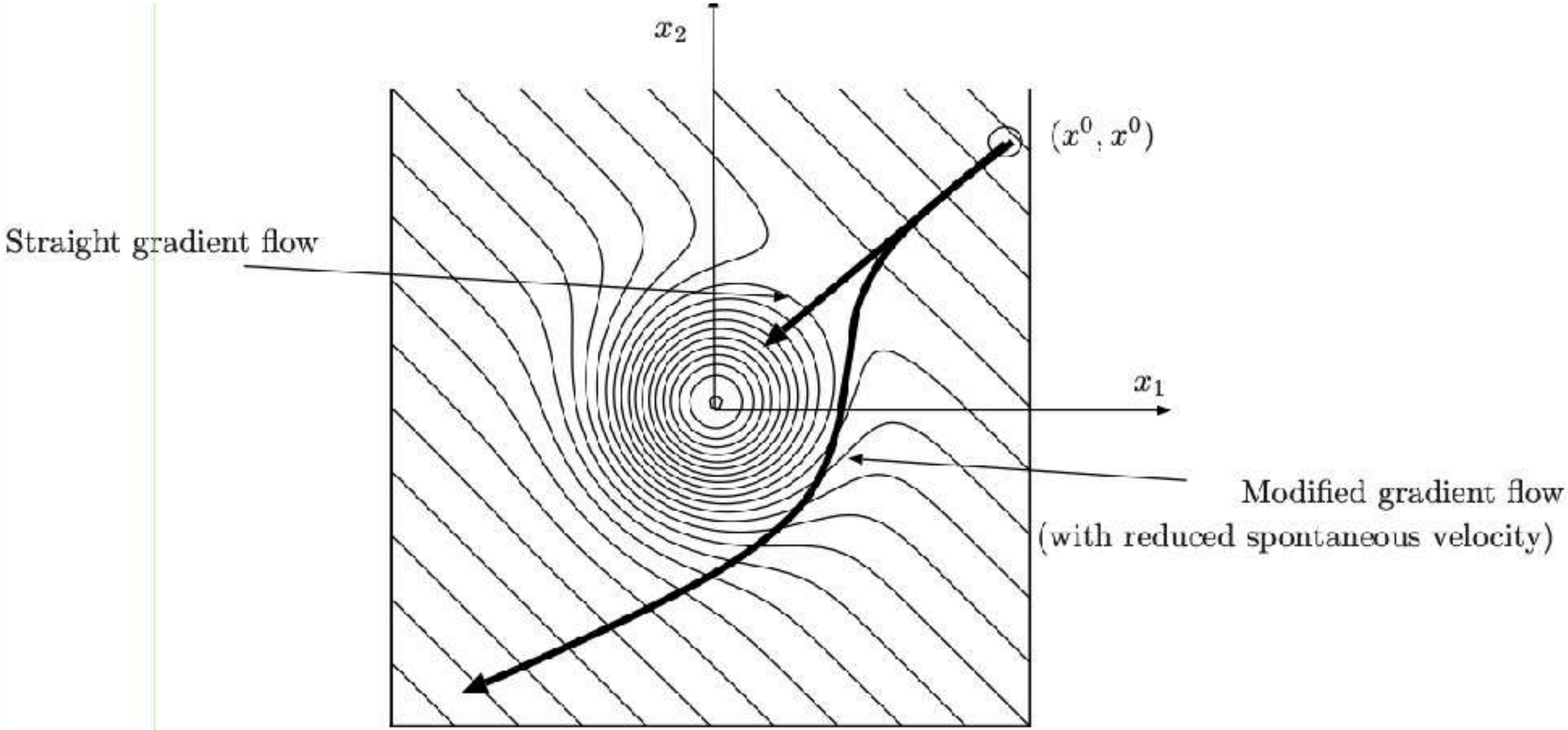
β_i : weight that *i* attributes to its own importance in the global dissatisfaction

One considers that when an individual cannot go forward because of someone upfront, they lower their β



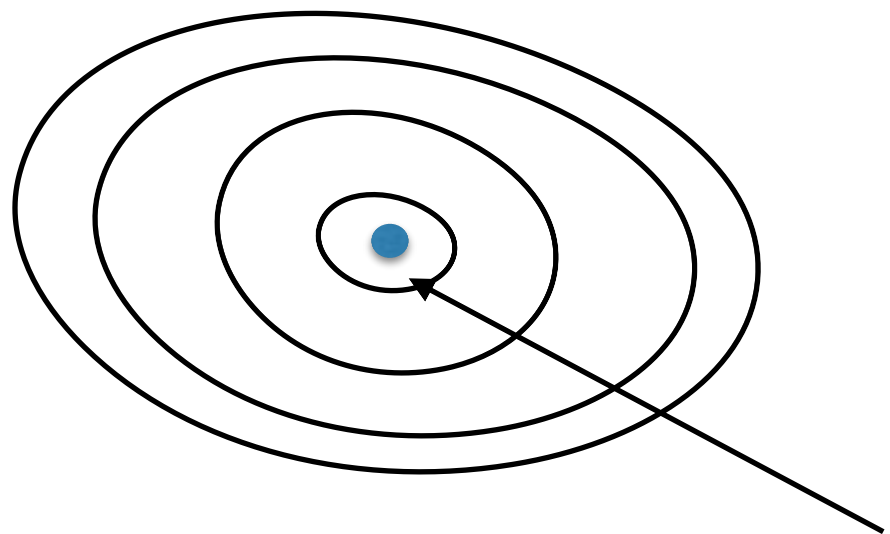
Faster is Slower Effect in crowd motion modeling

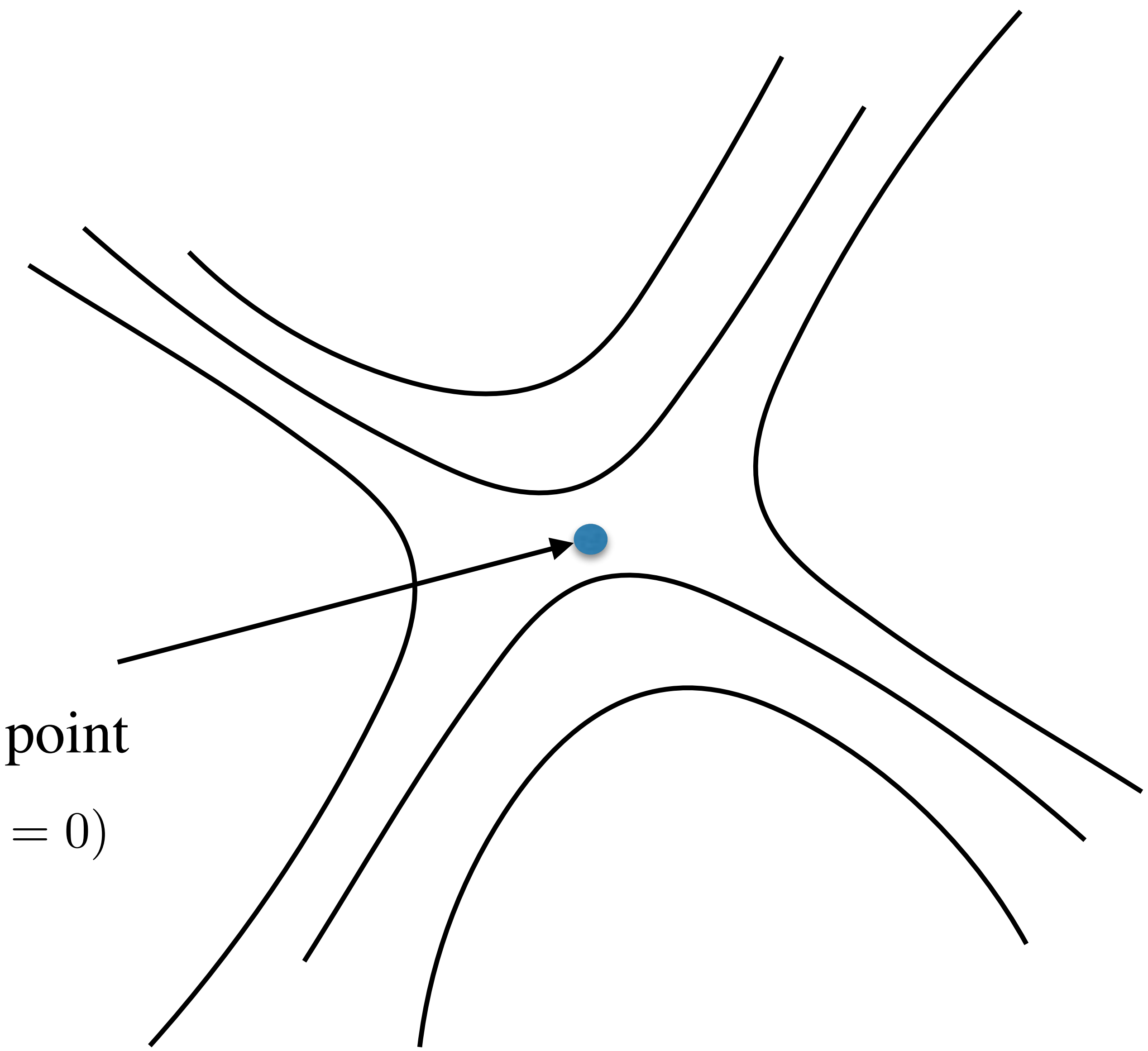
Steepest descent in a non-convex setting



Metastability / stability of jams

For a regular function $\dfrac{dq}{dt} = -\nabla\Psi(q)$





Critical point

$(\nabla\Psi(\bar{q}) = 0)$

At a critical point

$$\Psi(\bar{q}+h) = \Psi(\bar{q}) + \langle Hh|h\rangle + o(|h|^2)$$

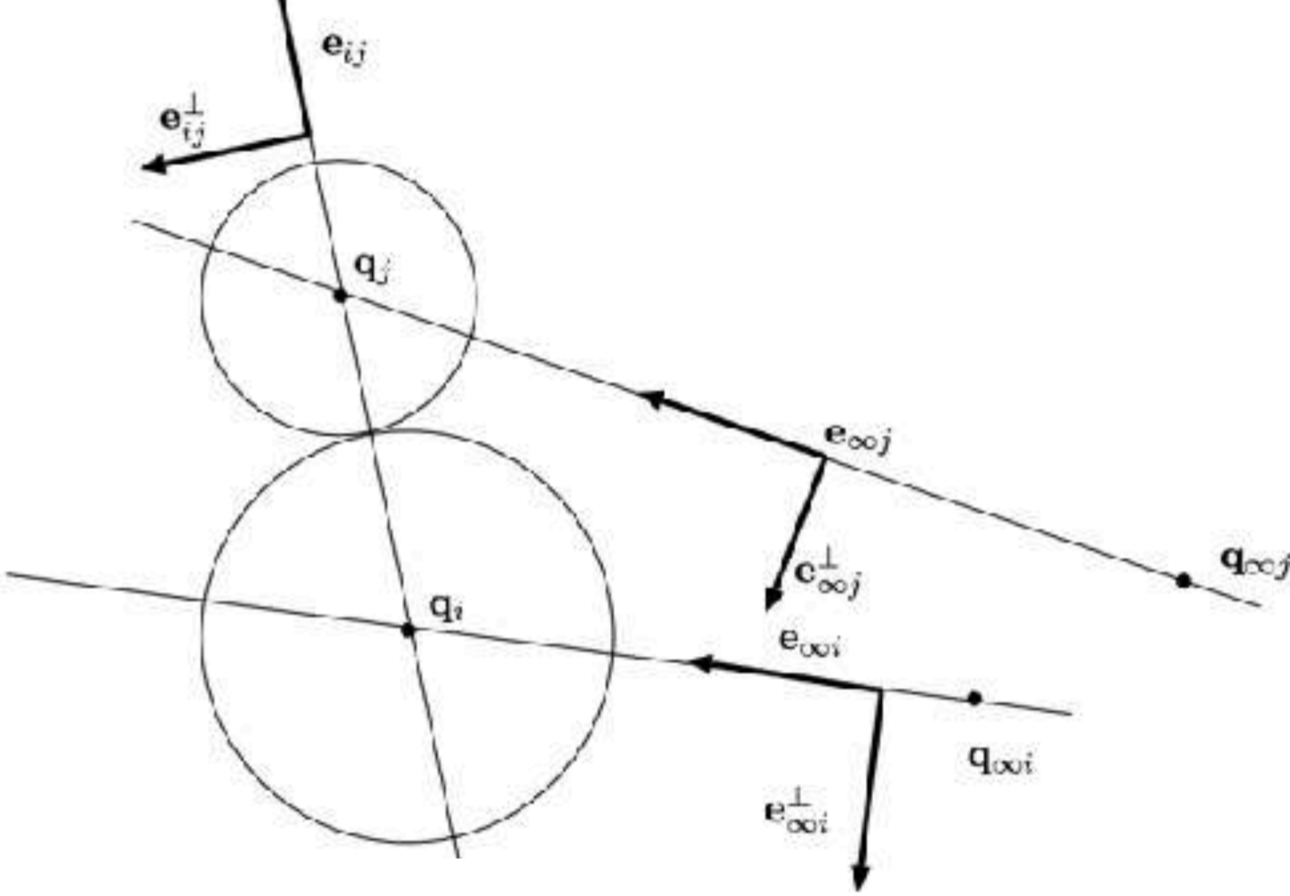
Stable equilibrium for the gradient flow (left), eigenvalues of H are > 0

Unstable equilibrium for the gradient flow (right), eigenvalues of H are of different signs

Metastability / stability of jams

Here: the function is highly non regular

Hessian-like matrix for a non smooth function



$$\Psi_\varepsilon \,:\, \mathbf{q} \in \mathbb{R}^{2N} \longmapsto \Psi_\varepsilon(\mathbf{q}) = \sum_{i=1}^{N} |\mathbf{q}_i - \mathbf{q}_{\infty i}| + \frac{1}{2\varepsilon}\sum_{i \neq j} (|\mathbf{q}_j - \mathbf{q}_i| - r_i - r_j)_-^2 .$$

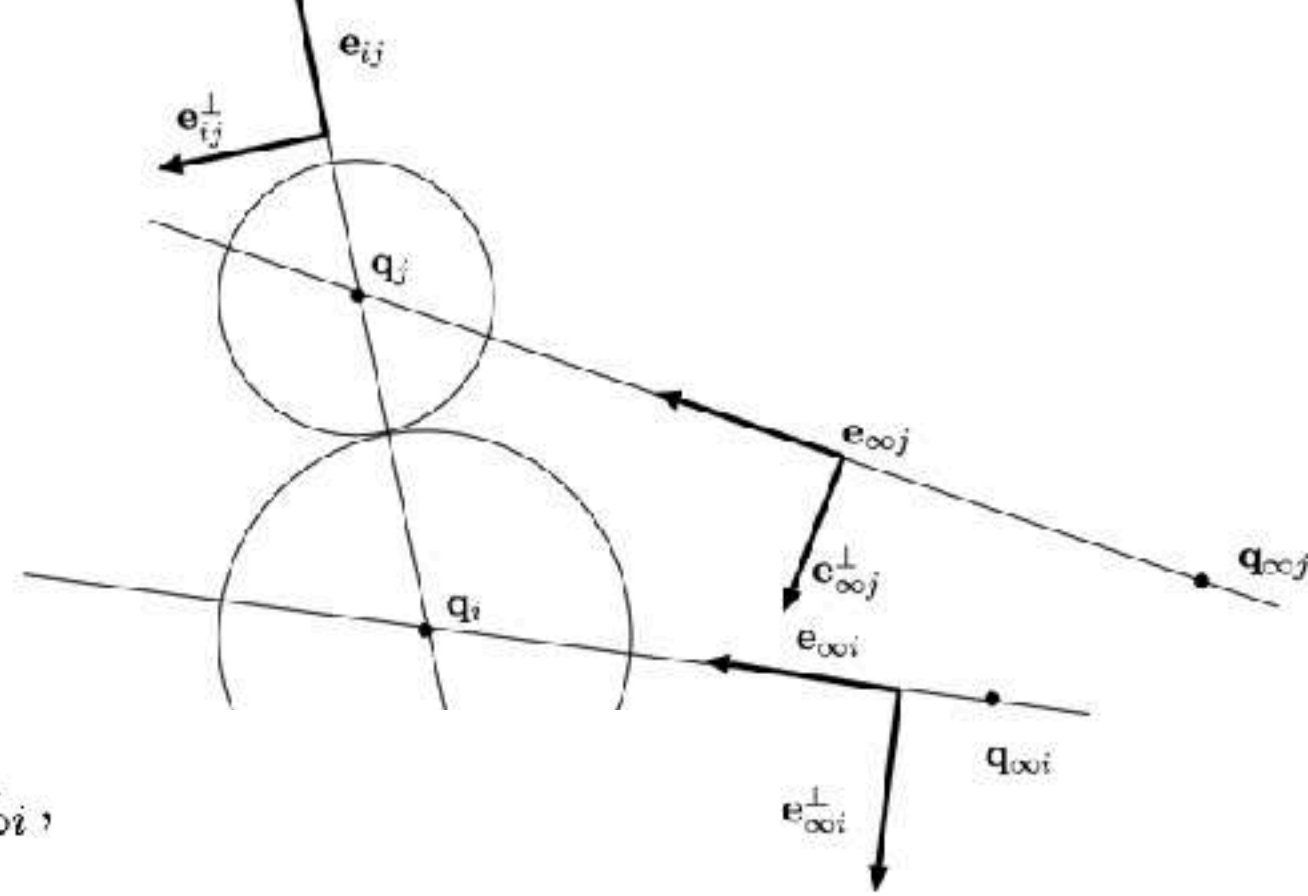
$$I_K(q) = \left| \begin{array}{ll} 0 & \text{if } q \in K \\ +\infty & \text{if } q \notin K \end{array} \right.$$

Overlapping penalty term

$$\nabla \Psi_\varepsilon = \sum_{i=1}^{N} \mathbf{e}_{\infty i}^{i} - \frac{1}{\varepsilon} \sum_{i \sim j} (r_i + r_j - |\mathbf{q}_j - \mathbf{q}_i|)(\mathbf{e}_{ij}^{j} - \mathbf{e}_{ij}^{i}).$$

$$\begin{aligned} \nabla_{\mathbf{q}_i} \left(\frac{1}{\varepsilon} (|\mathbf{q}_j - \mathbf{q}_i| - r_i - r_j)\, \mathbf{e}_{ij}^{i} \right) \;\; &= \;\; + \frac{1}{\varepsilon} \mathbf{e}_{ij}^{i} \otimes \mathbf{e}_{ij}^{i} \\ &\quad - \underbrace{\frac{1}{\varepsilon}(r_i + r_j - |\mathbf{q}_j - \mathbf{q}_i|)}_{\approx \lambda_{ij}} \frac{1}{|\mathbf{q}_i - \mathbf{q}_j|} \mathbf{e}_{ij}^{\perp i} \otimes \mathbf{e}_{ij}^{\perp i}. \end{aligned}$$

Metastability / stability of jams



$$H_\infty = \sum_i \frac{1}{|\mathbf{q}_i - \mathbf{q}_i^\infty|} \mathbf{e}_{\infty i}^{\perp} \otimes \mathbf{e}_{\infty i}^{\perp},$$

$$H_S = \sum_{i \sim j} \left(\mathbf{e}_{ij}^{i} \otimes \mathbf{e}_{ij}^{i} + \mathbf{e}_{ij}^{j} \otimes \mathbf{e}_{ij}^{j} - \mathbf{e}_{ij}^{i} \otimes \mathbf{e}_{ij}^{j} - \mathbf{e}_{ij}^{j} \otimes \mathbf{e}_{ij}^{i} \right)$$

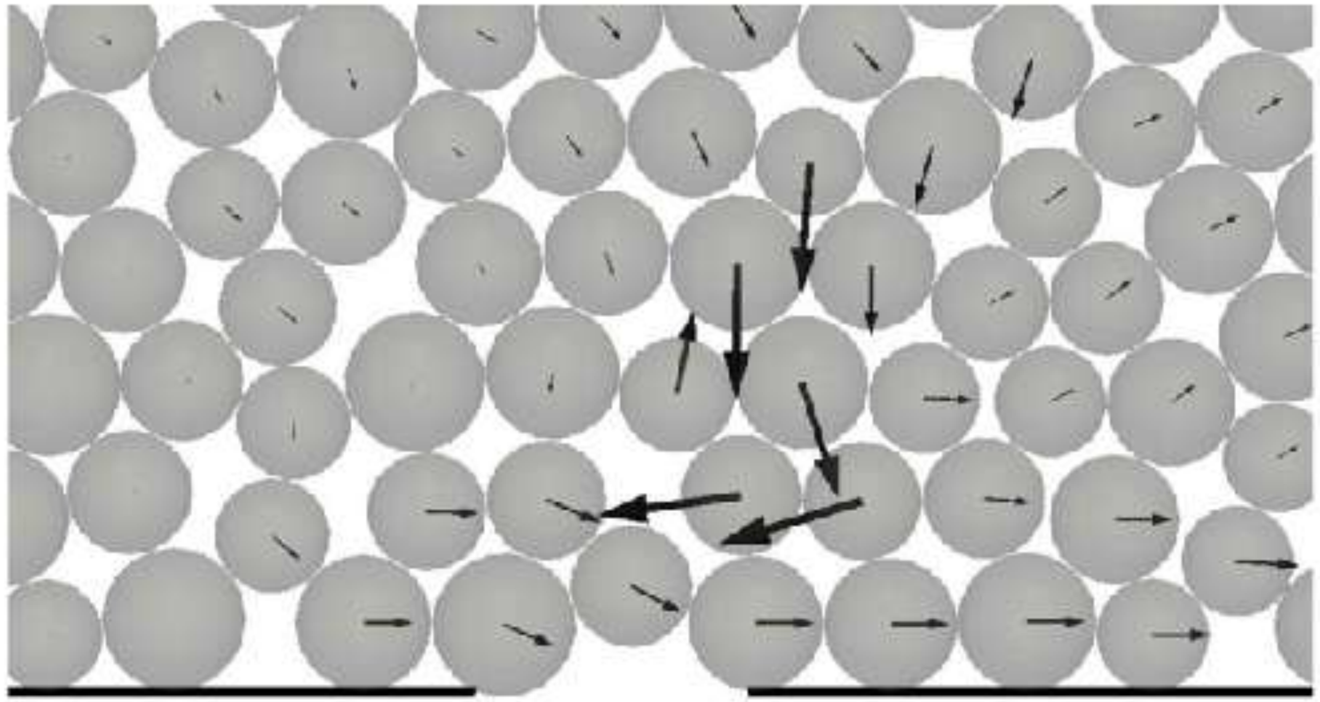
$$H_\lambda = -\sum_{i \sim j} \frac{\lambda_{ij}}{|\mathbf{q}_j - \mathbf{q}_i|} \left(\mathbf{e}_{ij}^{\perp i} \otimes \mathbf{e}_{ij}^{\perp i} + \mathbf{e}_{ij}^{\perp j} \otimes \mathbf{e}_{ij}^{\perp j} - \mathbf{e}_{ij}^{\perp i} \otimes \mathbf{e}_{ij}^{\perp j} - \mathbf{e}_{ij}^{\perp j} \otimes \mathbf{e}_{ij}^{\perp i} \right).$$

kernel of H_S :

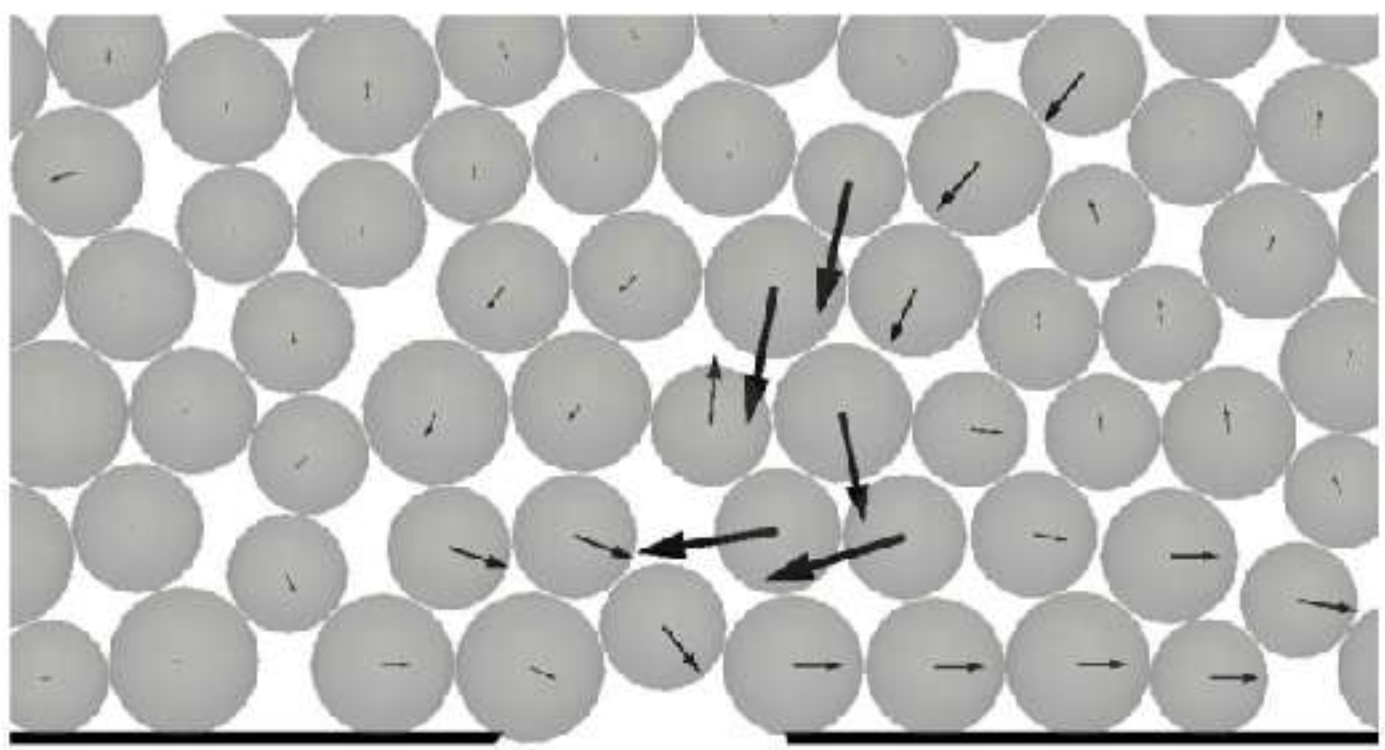
elementary displacement fields that do not change distances at the first order

Let $H^\star$ be the matrix that expresses the restriction to the kernel of H_S of the quadratic form associated to $H_\infty + H_\lambda$. Then $\mathbf{q}$ is stable whenever all eigenvalues of $H^\star$ are positive.

Quasi jams

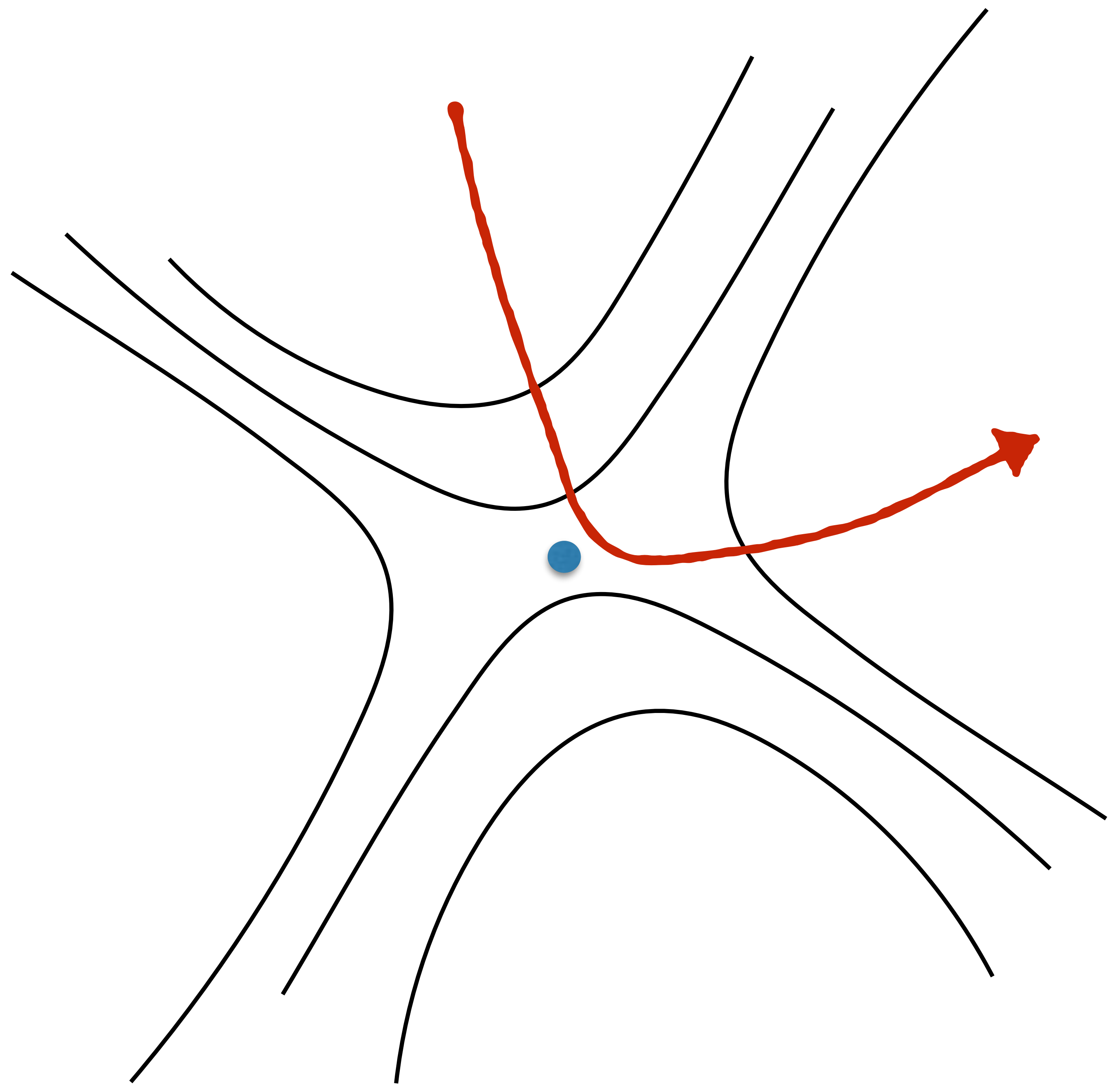


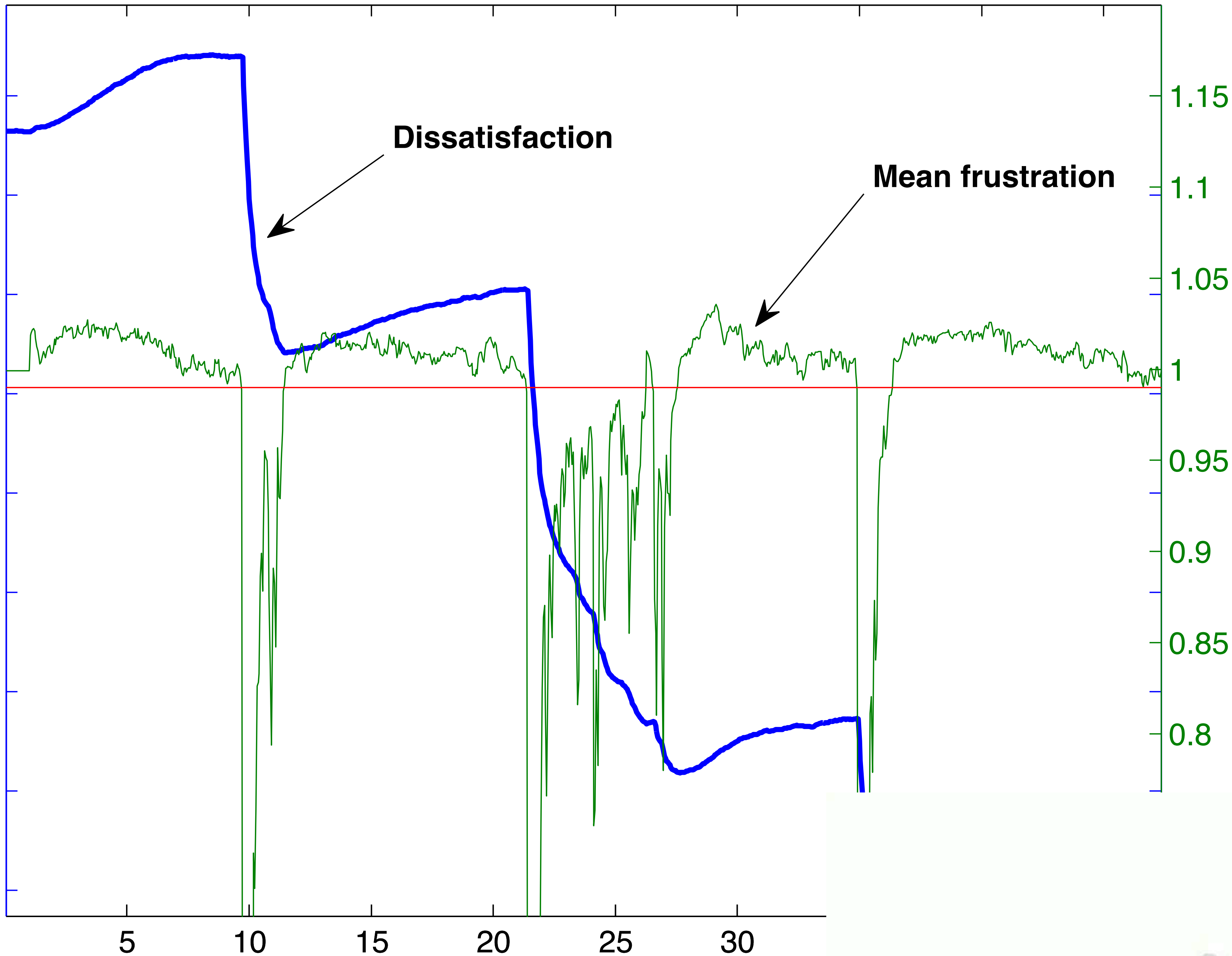
Actual velocity field



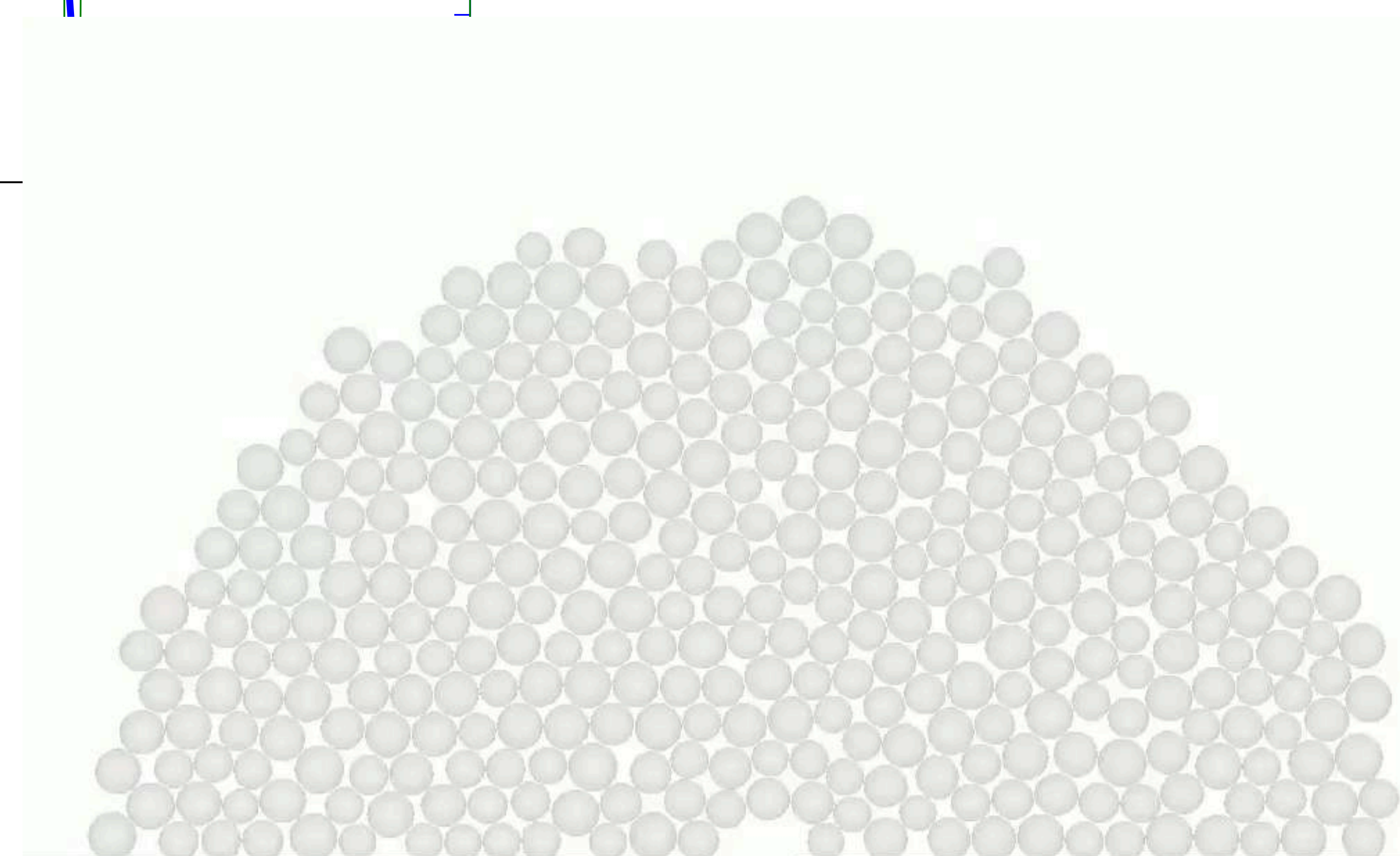
Eigenvector associated to the lowest eigenvalue

Quasi jams





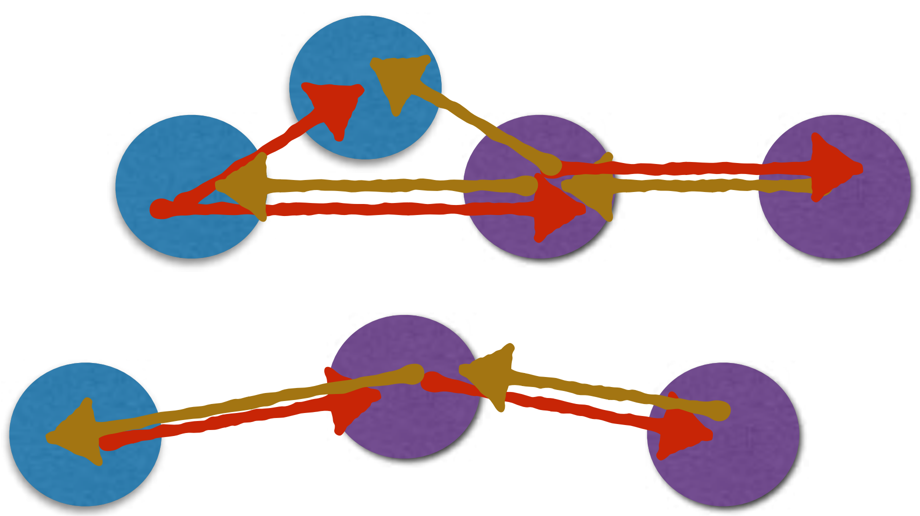
$$f_i = 1 - \frac{\mathbf{u}_i \cdot \mathbf{U}_i}{|\mathbf{U}_i|^2}.$$



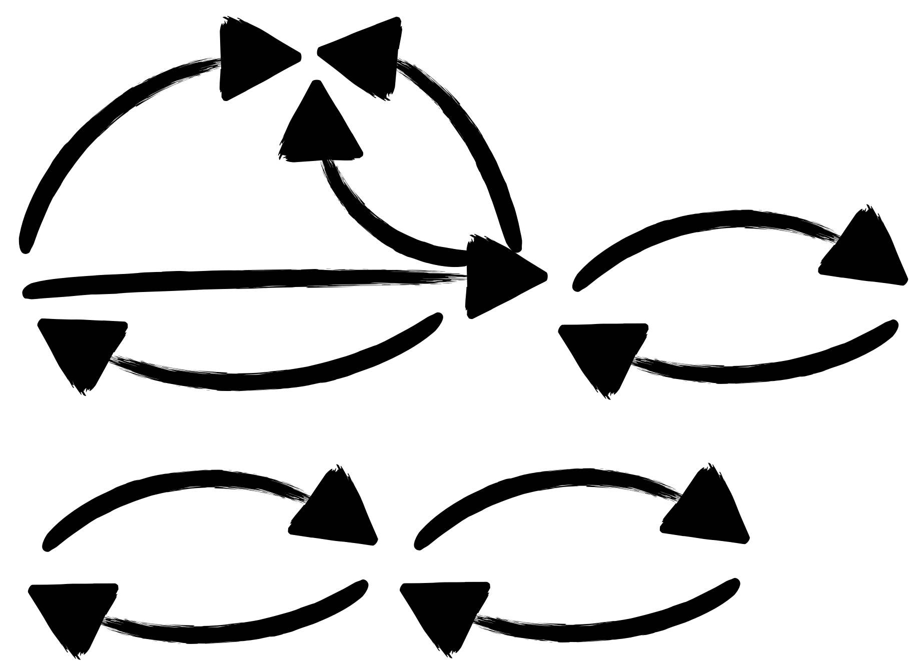
Nash-spirit approach

(with F. Al Reda)

Individual « sees » neighbors in I_i



Influence graph

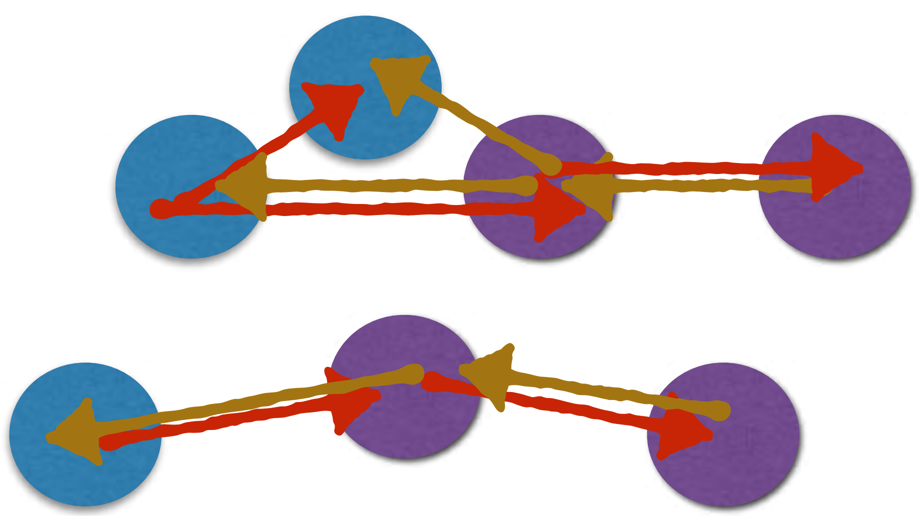


Each individual tends to realized their desired velocity, accounting for the non-overlapping contraint, given the other individual's velocities

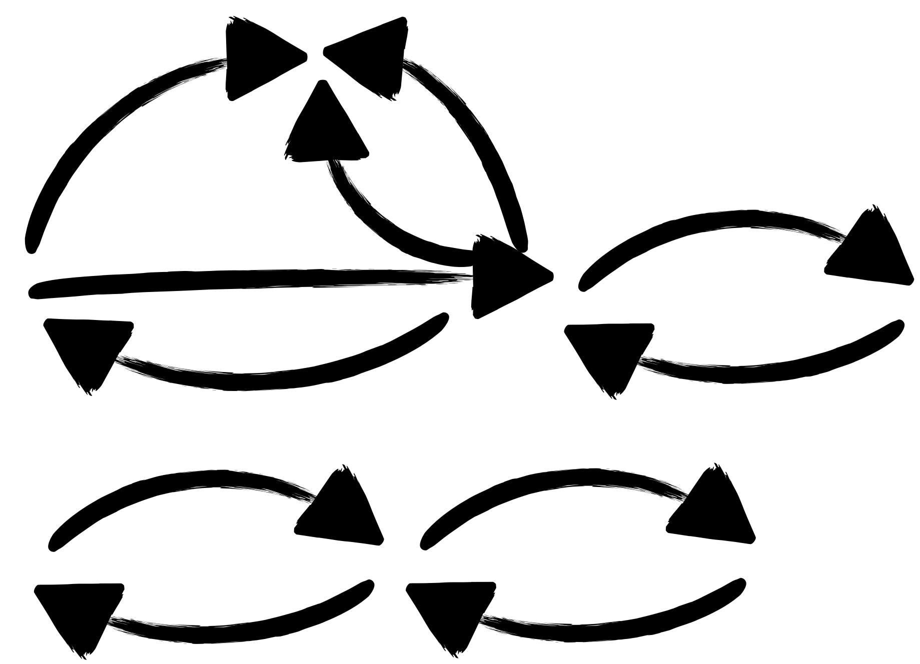
Nash-spirit approach

(with F. Al Reda)

Individual « sees » neighbors in I_i

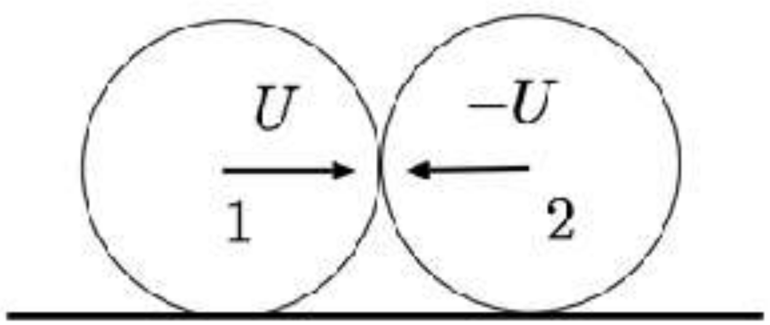


Influence graph

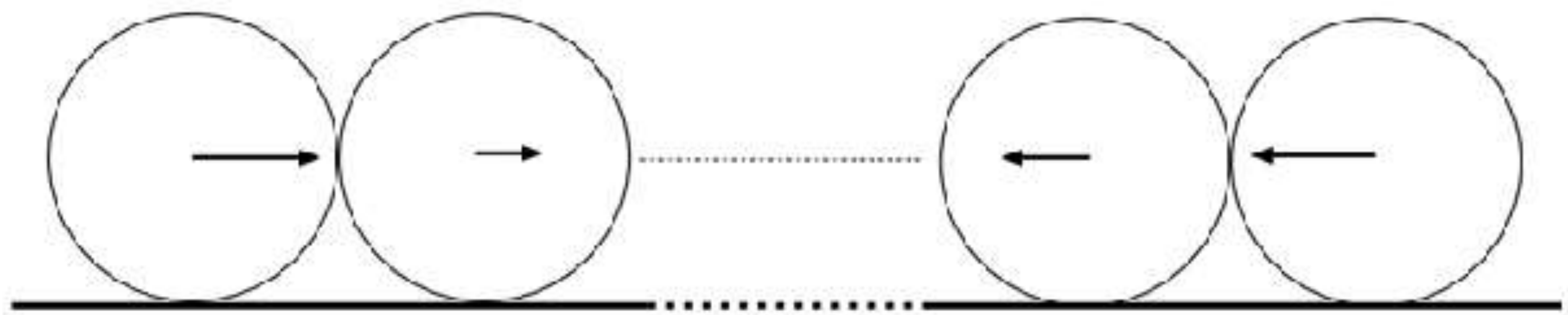


$$u_i = \underset{w \in C_i(q,u_{-i})}{\operatorname{argmin}} \frac{1}{2}|w - U_i|^2$$

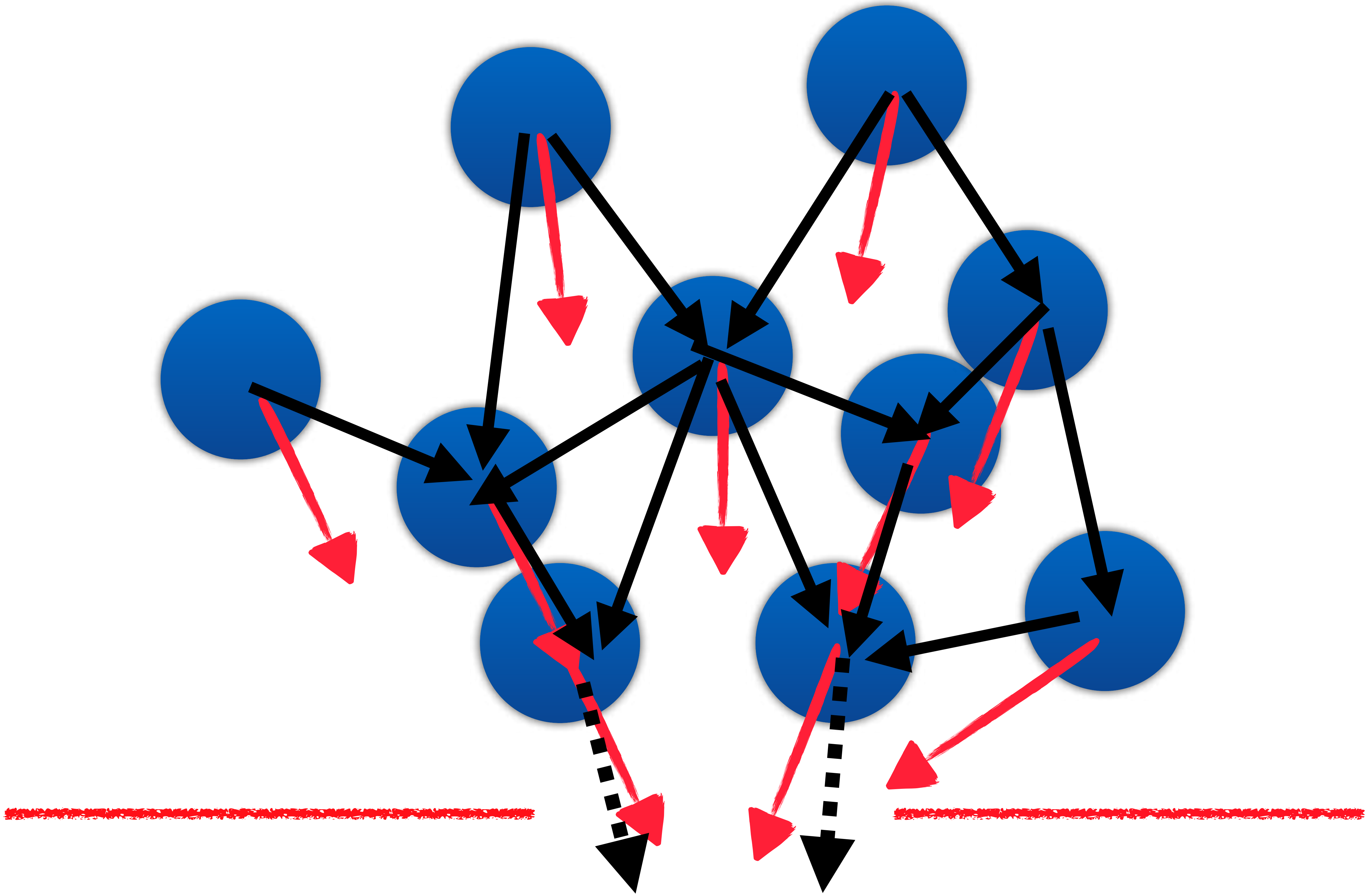
$$C_i(q, u_{-i}) = \left\{ w \in \mathbb{R}^d, \quad \forall j \in I_i, \quad D_{ij}(q) = 0 \Rightarrow e_{ij}(q) \cdot (w - u_j) \leq 0 \right\}$$

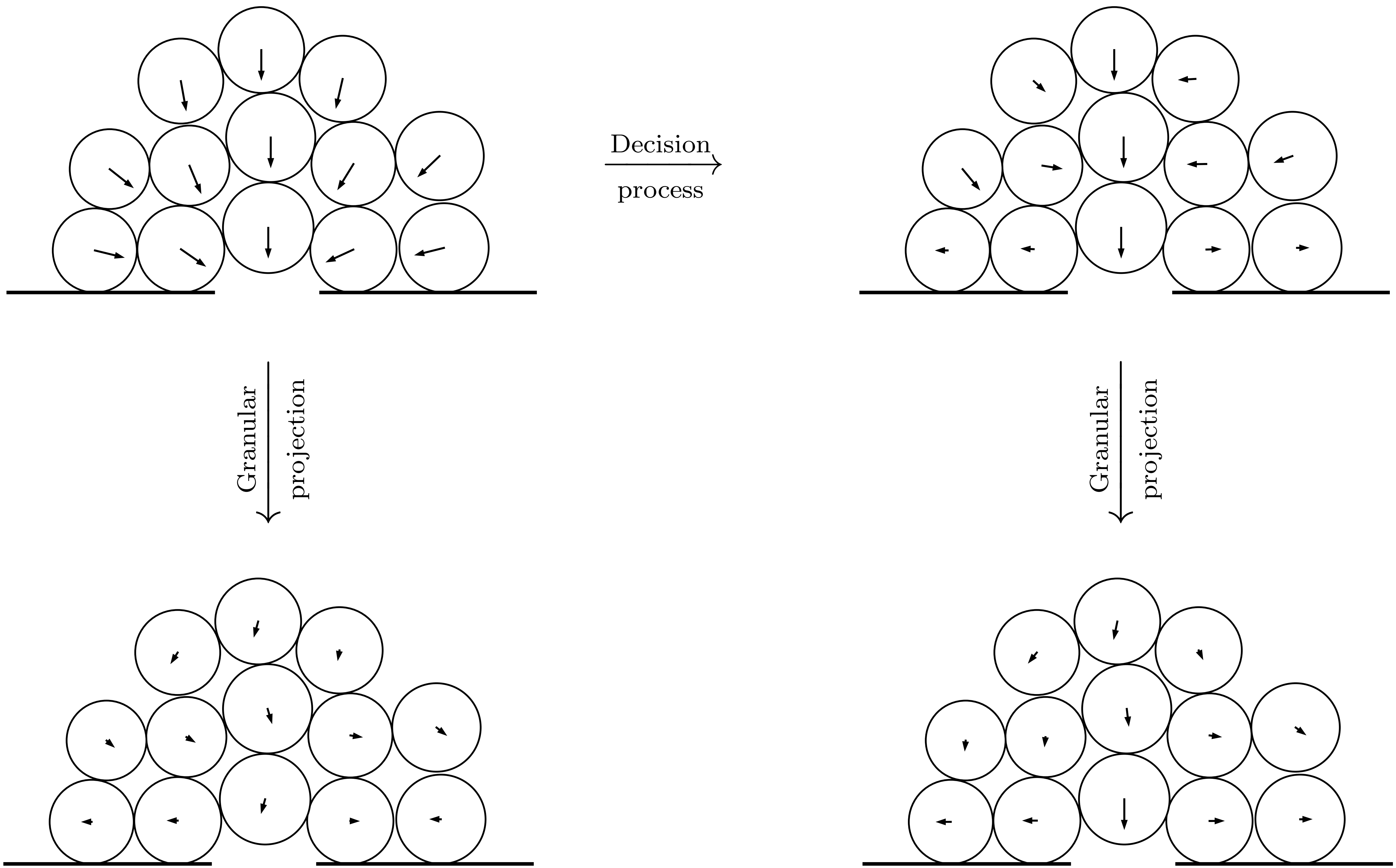


U
$-U$
1
2

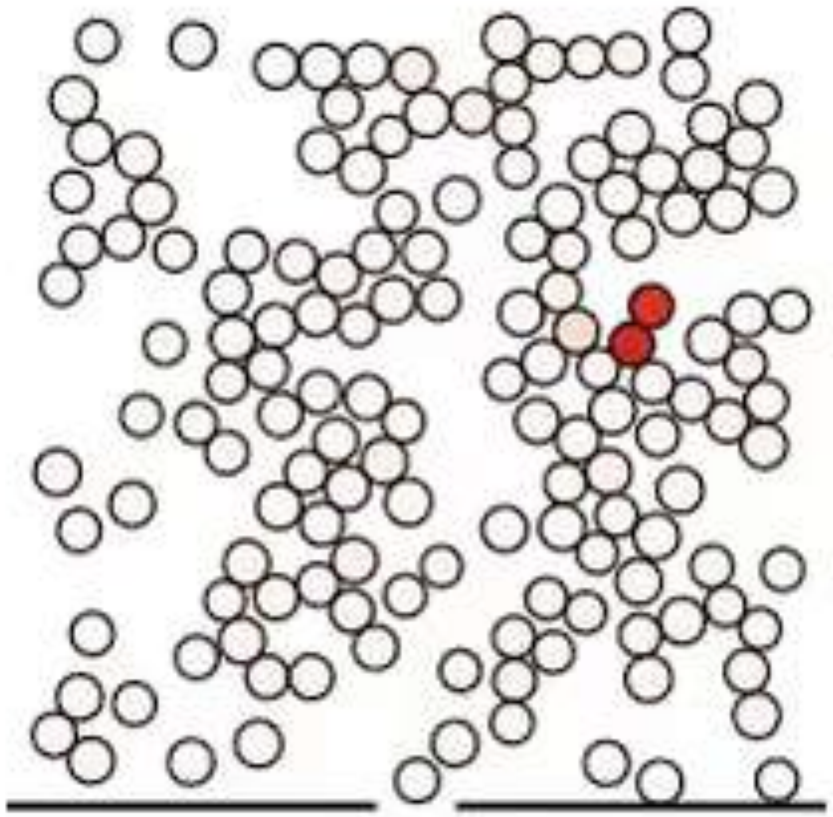


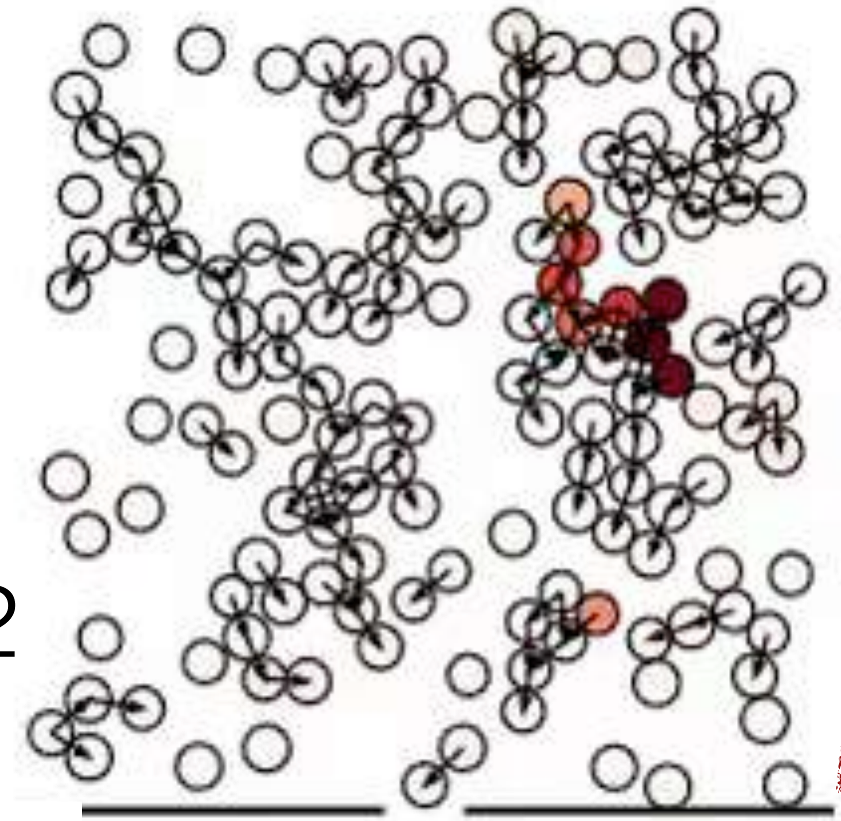
Hierarchical situation (acyclical)

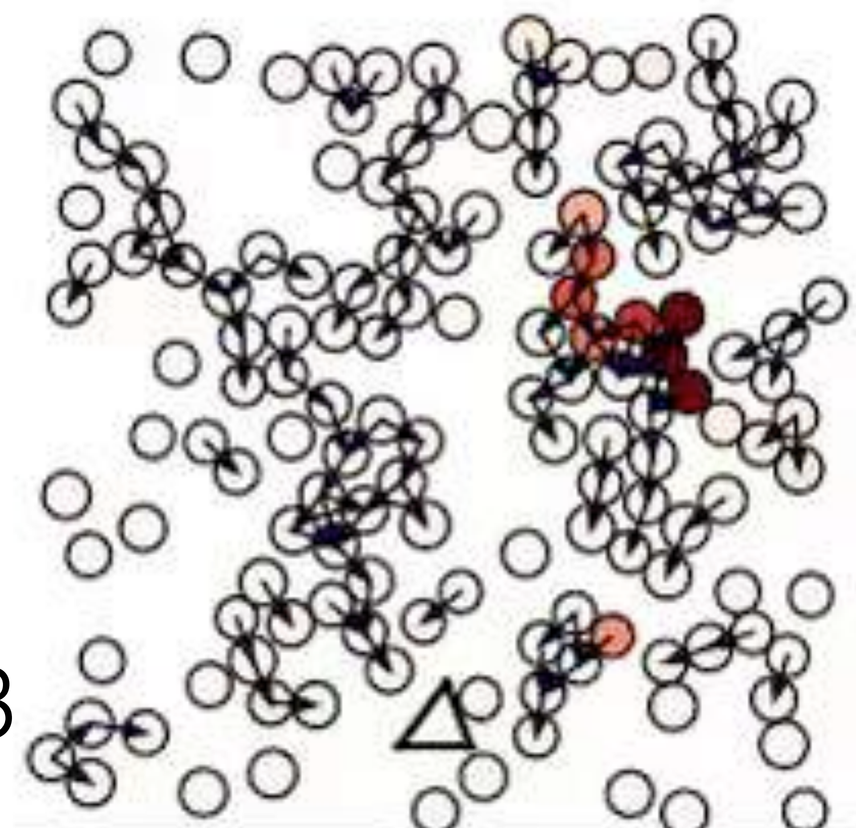



Decision
process
Granular
projection
Granular
projection

Inhibition based model (with F. Al Reda)





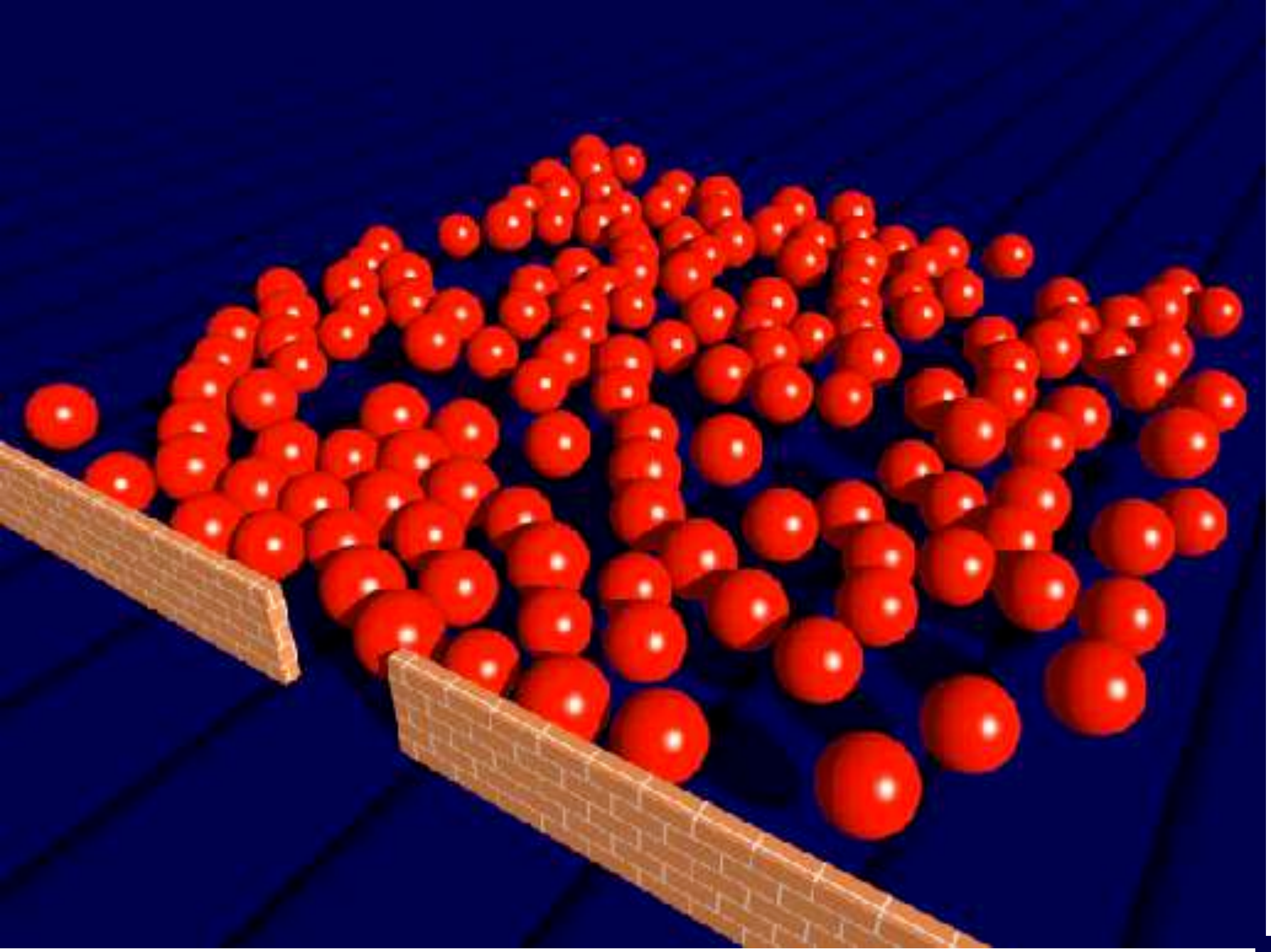


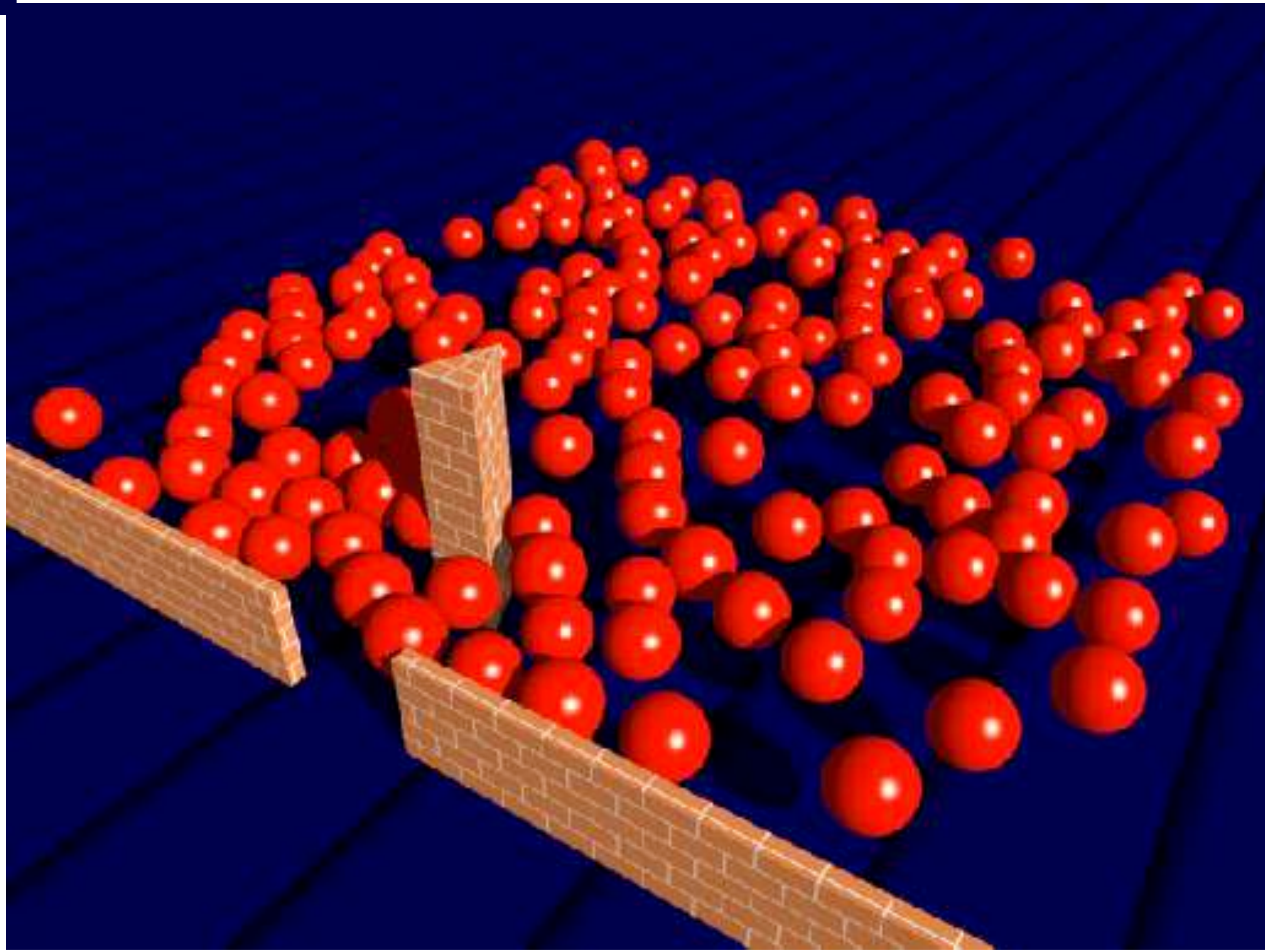
Selfish model

Individuals do not push (reduction of the velocity)

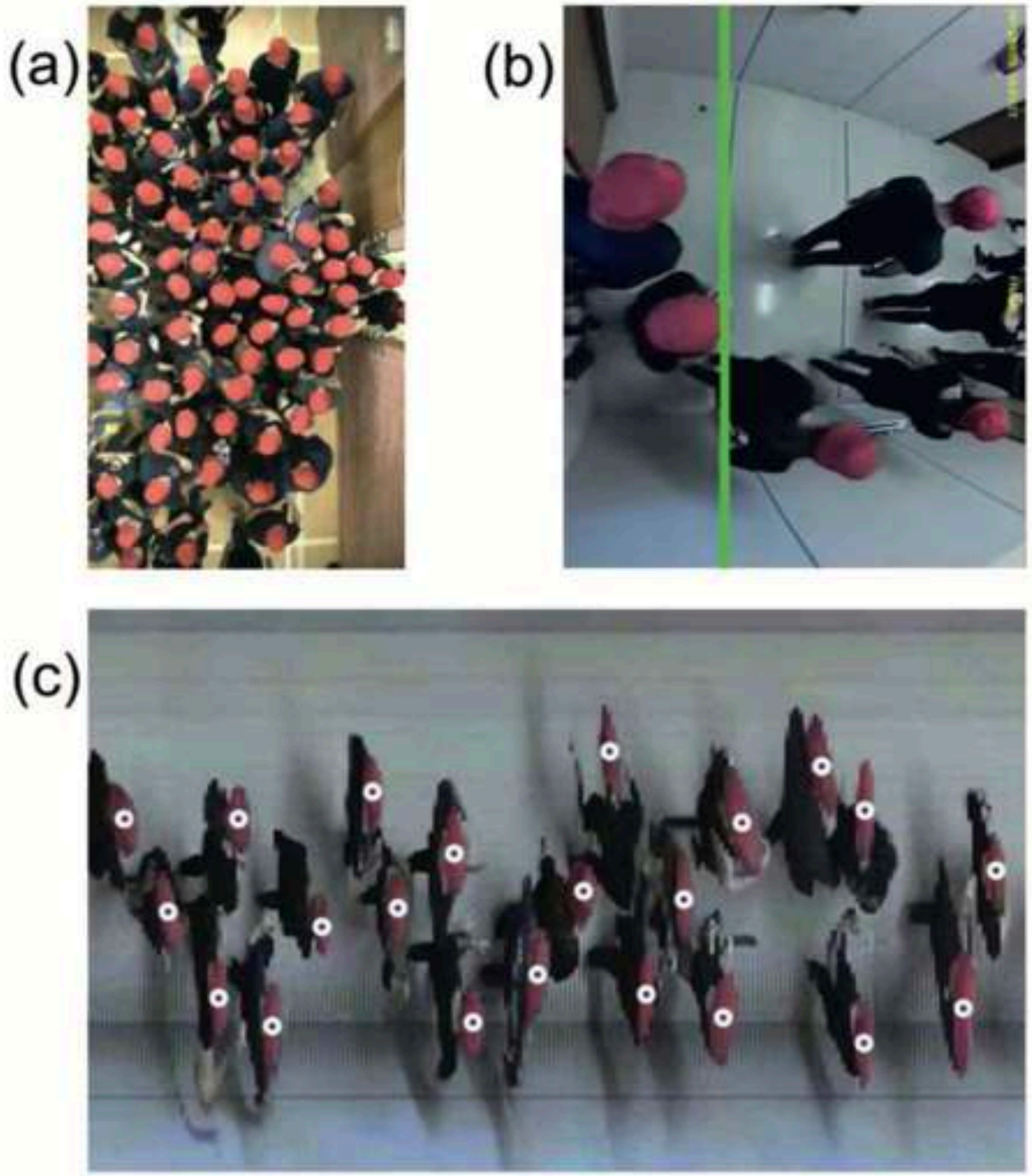
Same with an obstacle

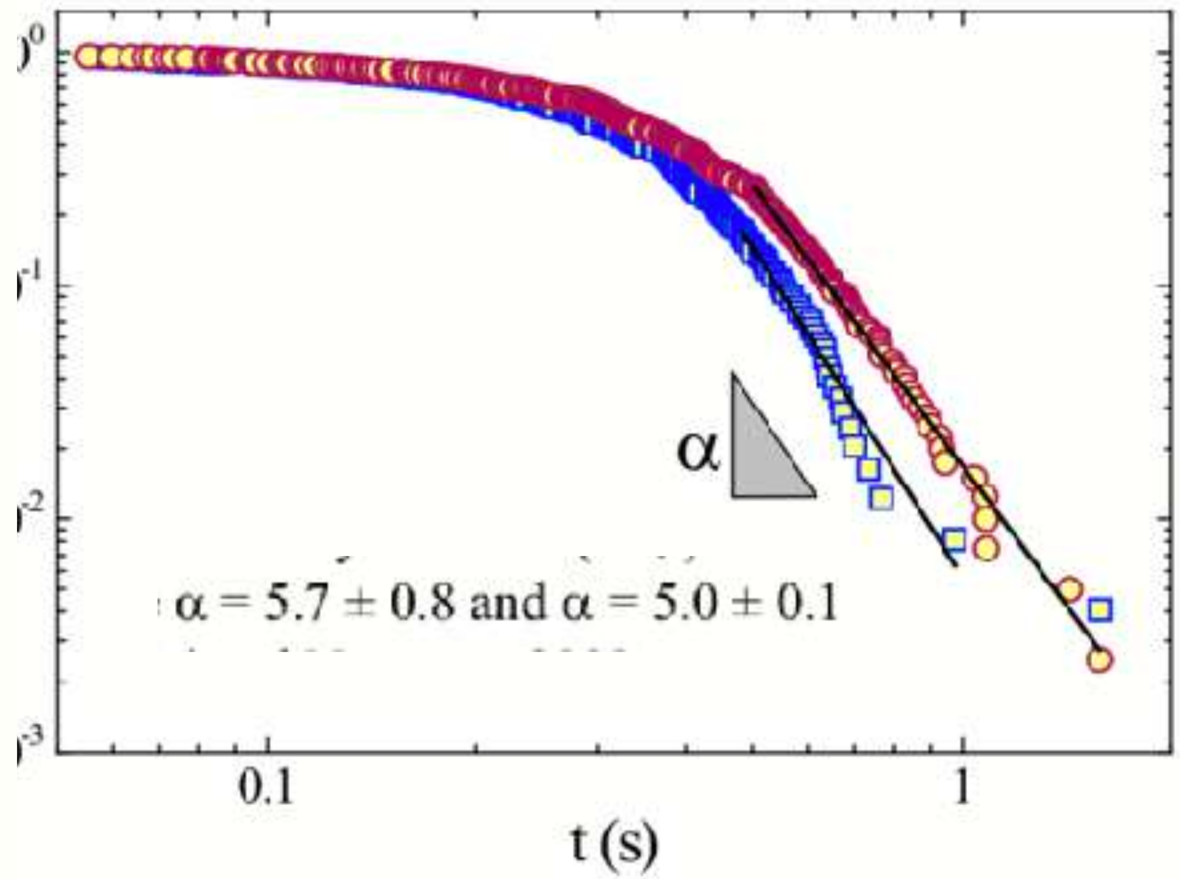
Computations: F. Al Reda





Validation



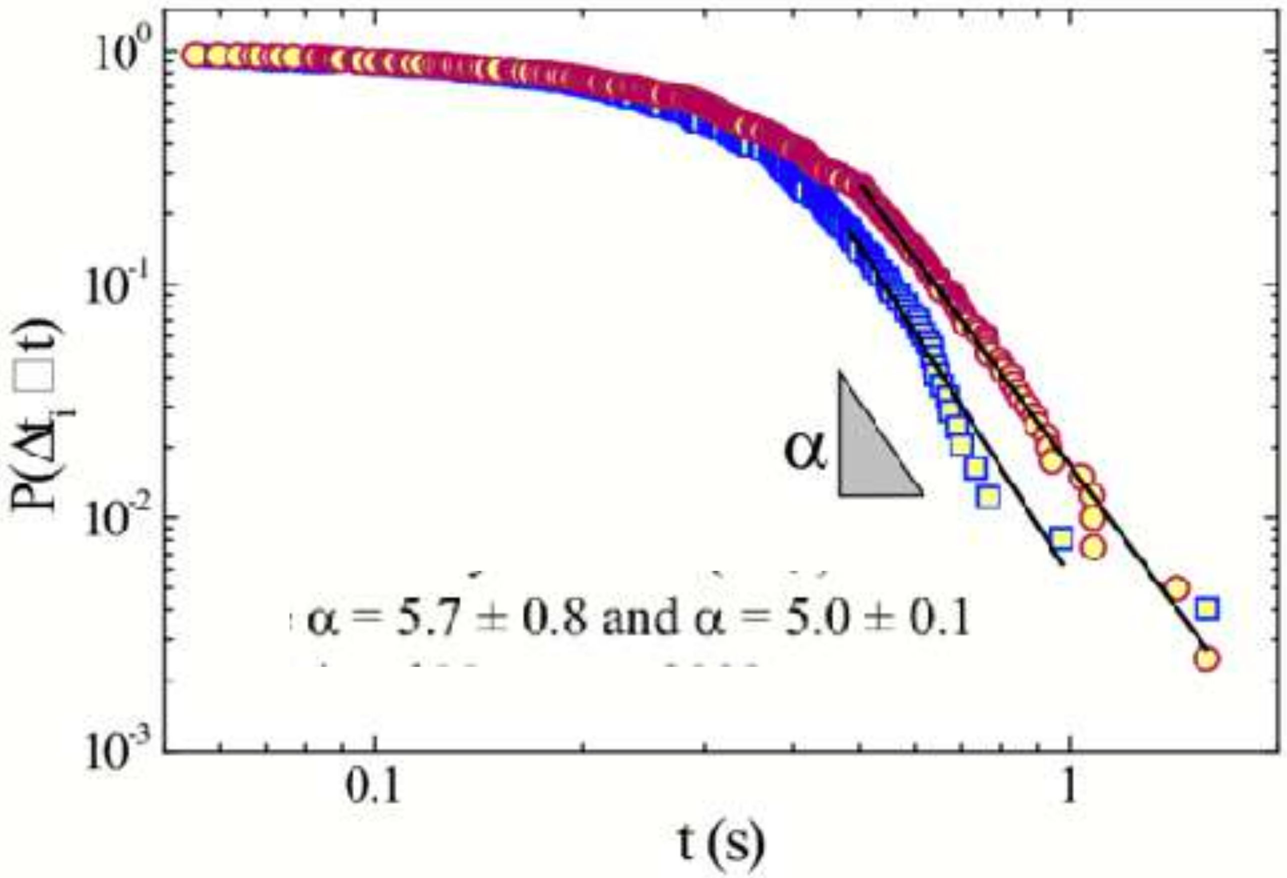


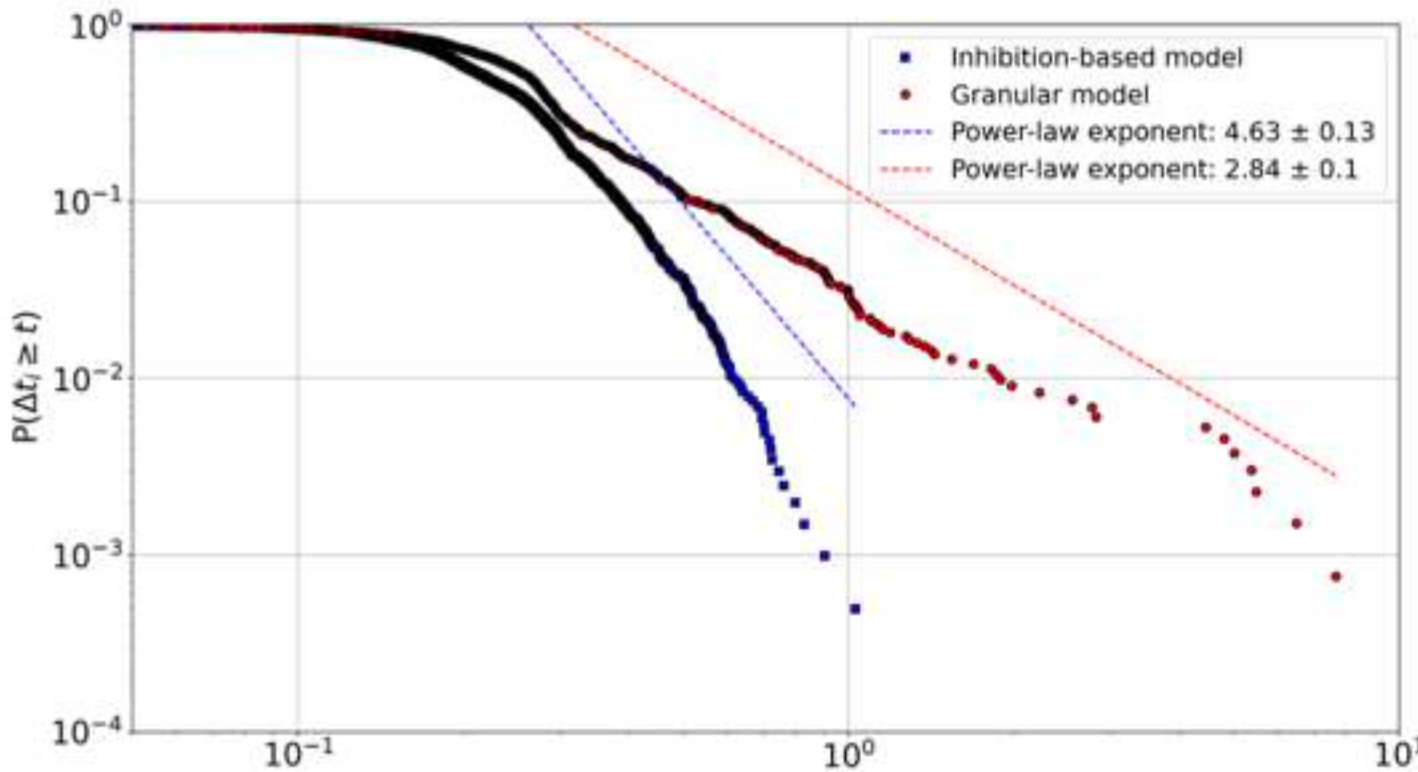
Experimental evidence of the "Faster Is Slower" effect

A. Garcimartín [a,*], I. Zuriguel [a], J.M. Pastor [a], C. Martín-Gómez [b], D.R. Parisi [c]

Validation

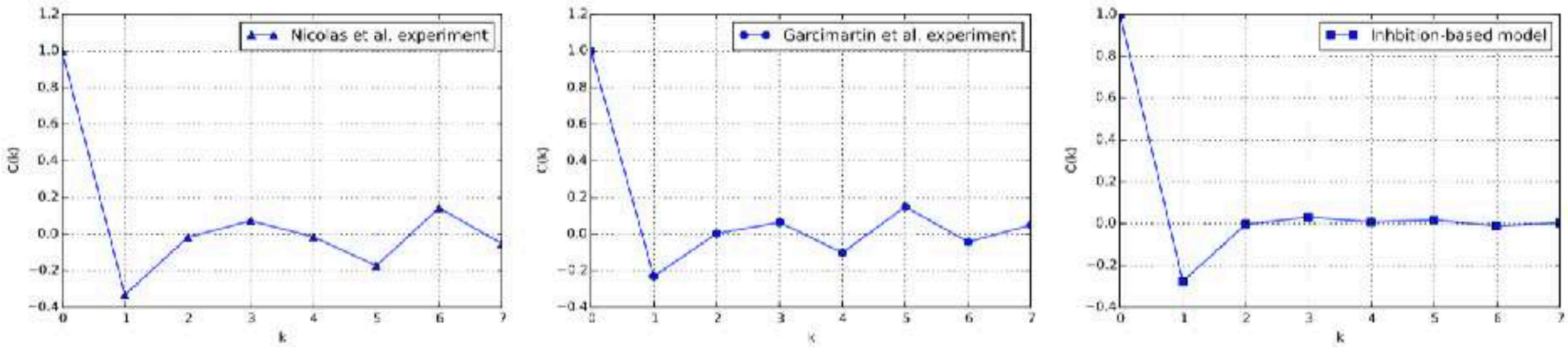
Power law distribution & Faster is Slower effect





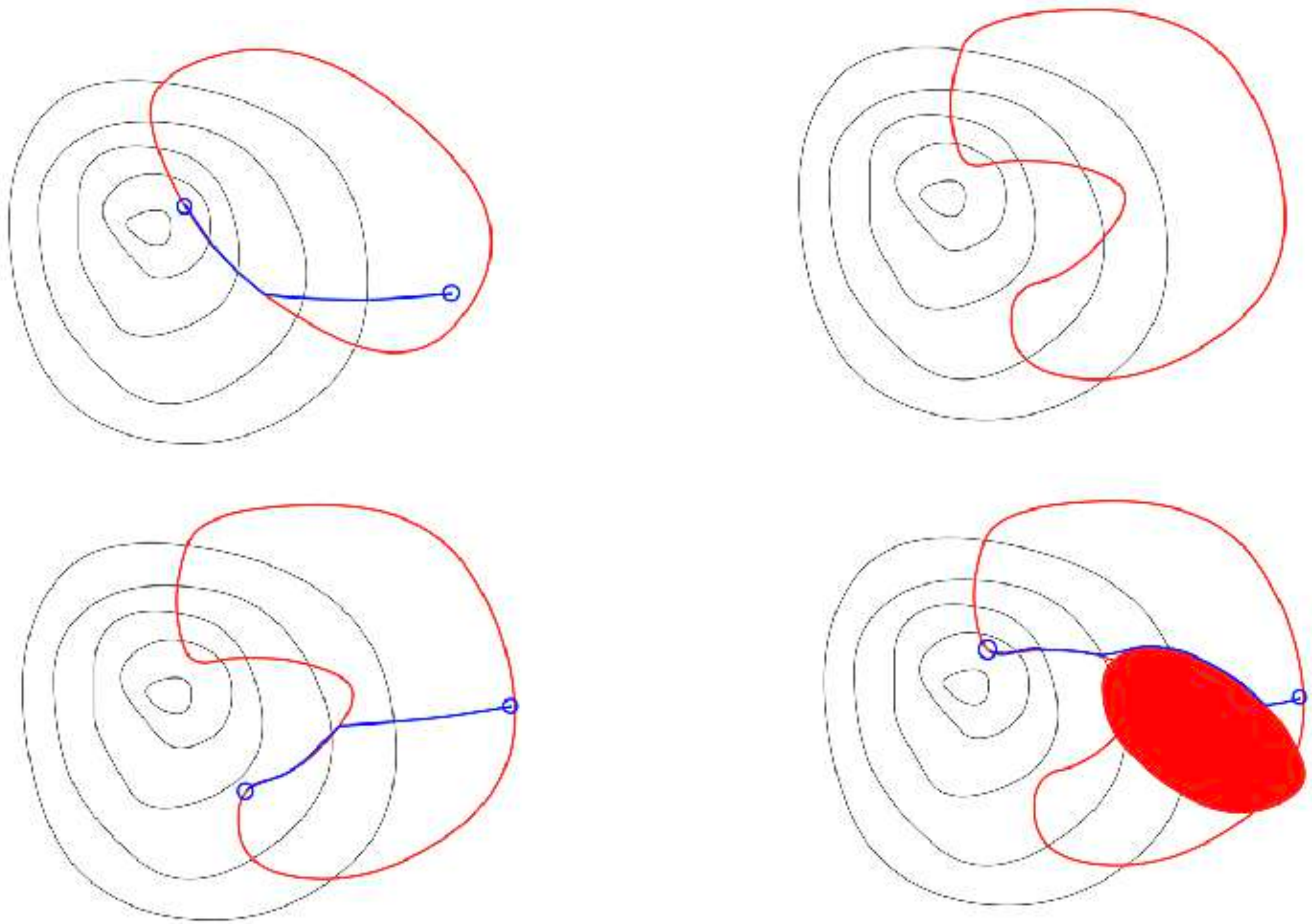
Validation

Alternation between short and long time lapses

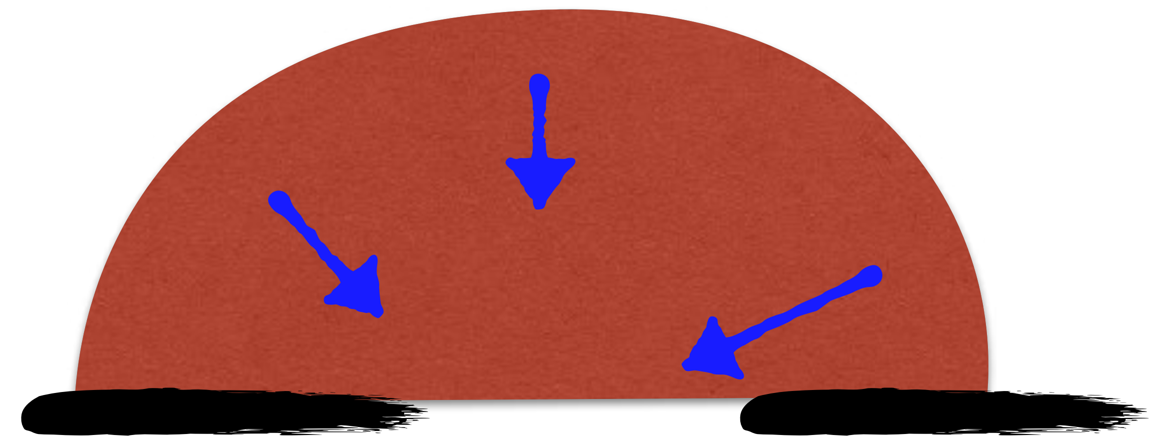


Nicolas, A., Bouzat, S., Kuperman, M.N., 2017. Pedestrian flows through a narrow doorway: Effect of individual behaviours on the global flow and microscopic dynamics. Transportation Research Part B: Methodological 99, 30–43.

Role of an obstacle



Macroscopic setting



Spontaneous velocity $U = U(x)$

Feasible densities $K = \{\rho \in \mathcal{P}(\Omega)\ ,\ \ 0 \le \rho \le 1 \quad a.e.\}$

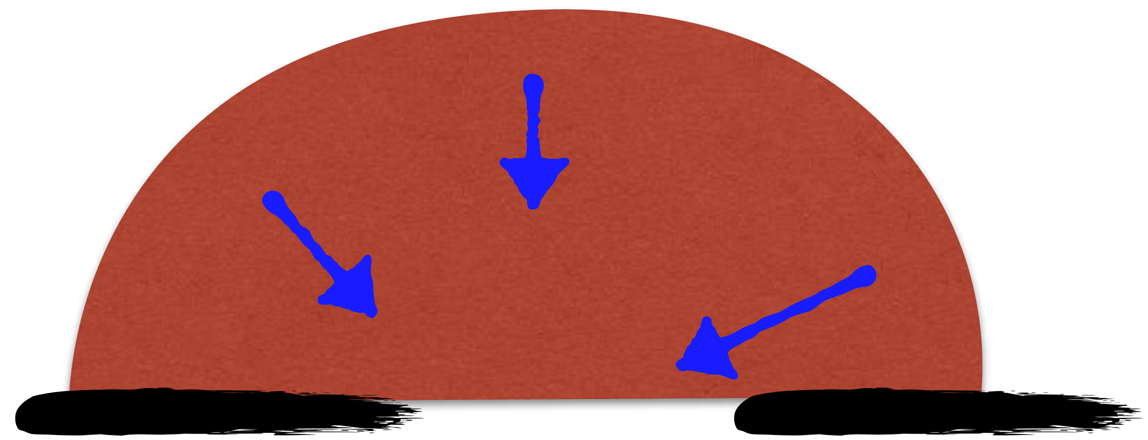
$$\left| \begin{array}{r} \dfrac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0 \\ u = P_{C_\rho} U \end{array} \right.$$

Cone of feasible velocities : nonnegative divergence wherever $\rho = 1$

$$C_\rho = \left\{ v \in L^2_\rho(\Omega)\ ,\ \ \int_\Omega v \cdot \nabla p \le 0 \quad \forall p \in H^1_\rho\ ,\ \ p \ge 0 \quad a.e. \right\}$$

$$H^1_\rho = \{p \in H^1(\Omega)\ ,\ \ p(1-\rho) = 0\ a.e.\}$$

Macroscopic setting



Theorerical framework: gradient flow structure in the Wasserstein space (Optimal Transportation)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0$$

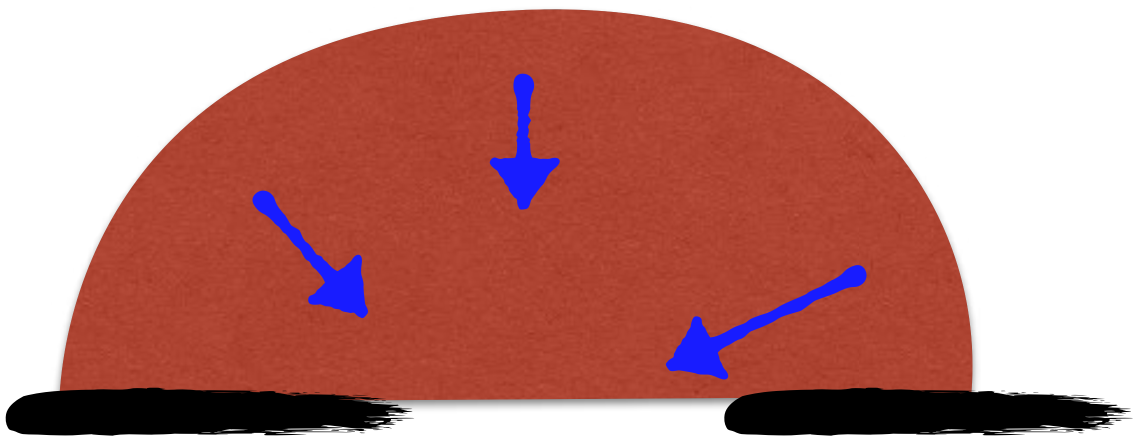
$$u \in -\partial(J(\rho) + I_K(\rho)) \qquad J(\rho) := \int_\Omega D(x)\rho(x)\,dx$$

J.K.O Scheme (Jordan Kinderlehrer Otto) :

$$\begin{cases} \rho_\tau^0 & = & \rho^0 \\ \rho_\tau^k & \in & \underset{\mathcal{P}_2(\mathbb{R}^d)}{\operatorname{argmin}} \left\{ J(\rho) + \mathbf{I}_K(\rho) + \frac{1}{2\tau} W_2^2(\rho, \rho_\tau^{k-1}) \right\} \end{cases}$$

A macroscopic crowd motion model of gradient flow type, M3AS, 2010, B.M., A. Roudneff Chupin, F. Santambrogio

Macroscopic setting



$$u = P_{C_\rho} U$$

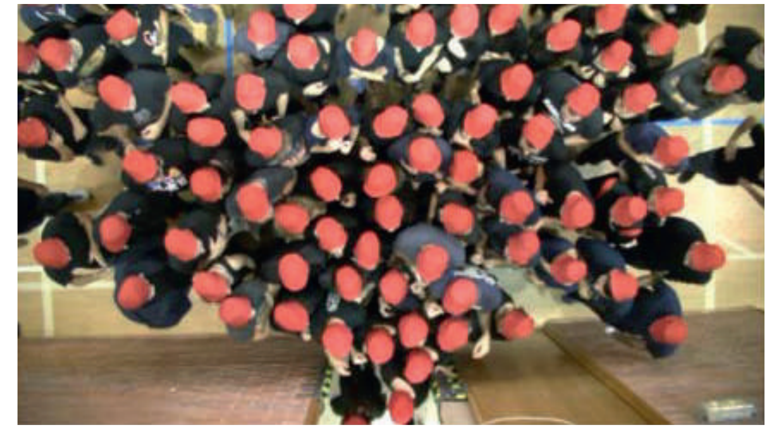
$$C_\rho = \left\{ v \in L^2_\rho(\Omega)\,,\ \ \int_\Omega v \cdot \nabla p \le 0 \quad \forall p \in H^1_\rho\,,\ \ p \ge 0 \quad a.e. \right\}$$

$$H^1_\rho = \{ p \in H^1(\Omega)\,,\ \ p(1-\rho) = 0 \; a.e.\}$$

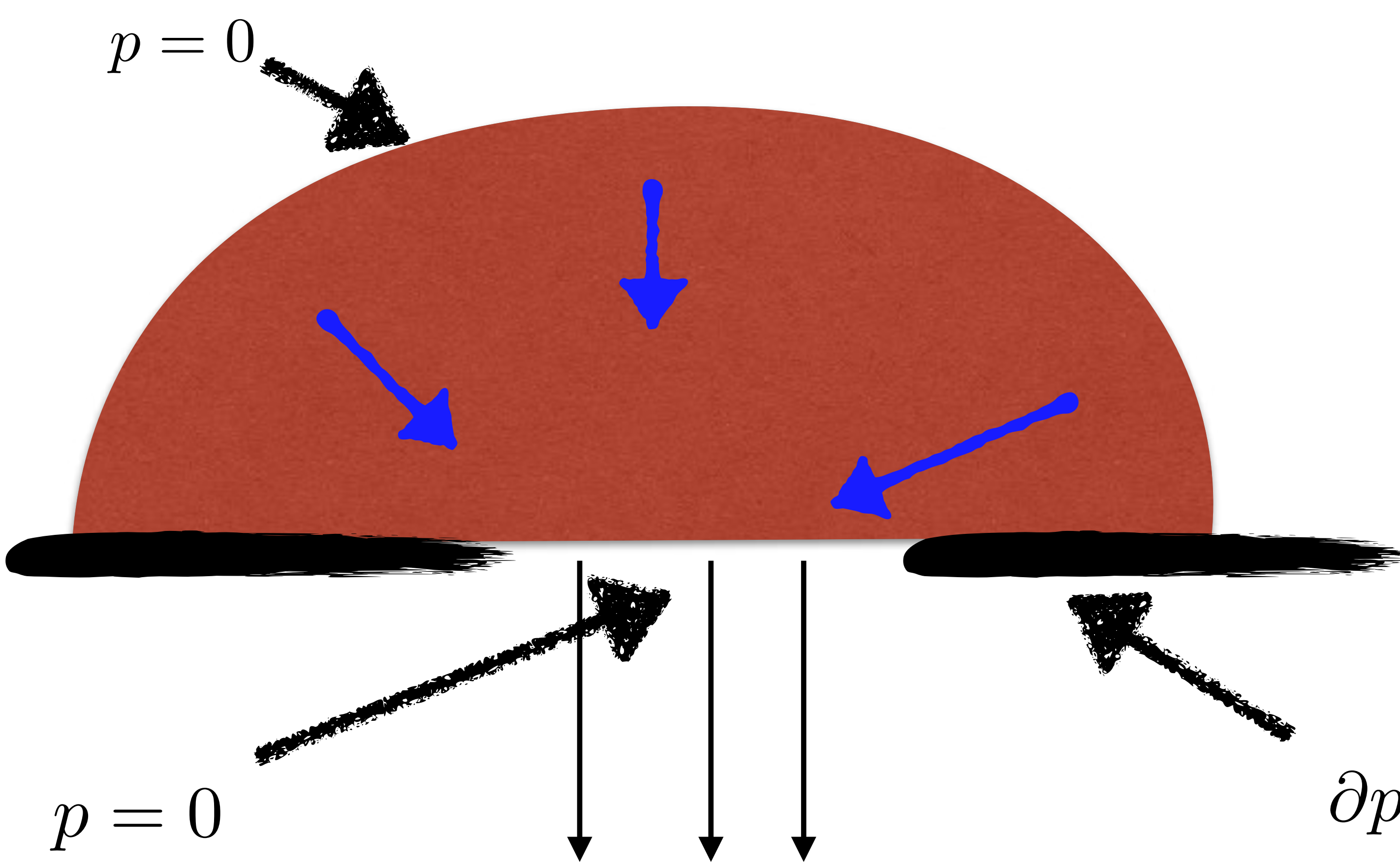
Unilateral Darcy problem :

$$\left| \begin{array}{lcl} u + \nabla p & = & U \\ -\nabla \cdot u & \le & 0 \\ p & \ge & 0 \\ \displaystyle\int_\omega u \cdot \nabla p & = & 0, \end{array} \right.$$

Evacuation of a room



$$u \quad = \quad P_{C_\rho} U$$



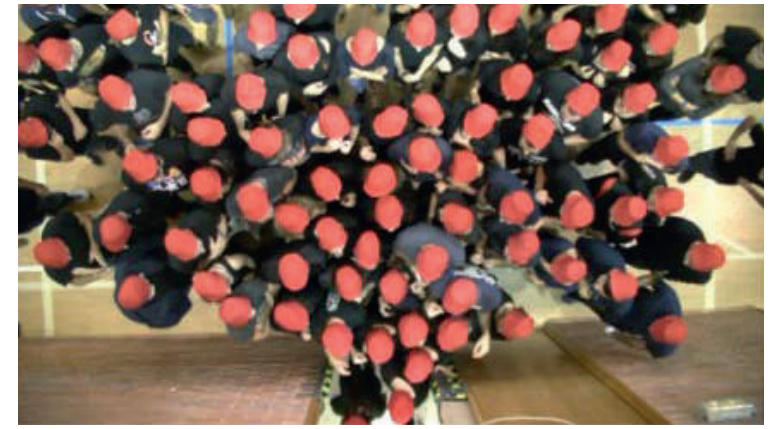
$$\left| \begin{array}{rcl} u + \nabla p & = & U \\ -\nabla \cdot u & \leq & 0 \\ p & \geq & 0 \\ \displaystyle\int_\omega u \cdot \nabla p & = & 0, \end{array} \right.$$

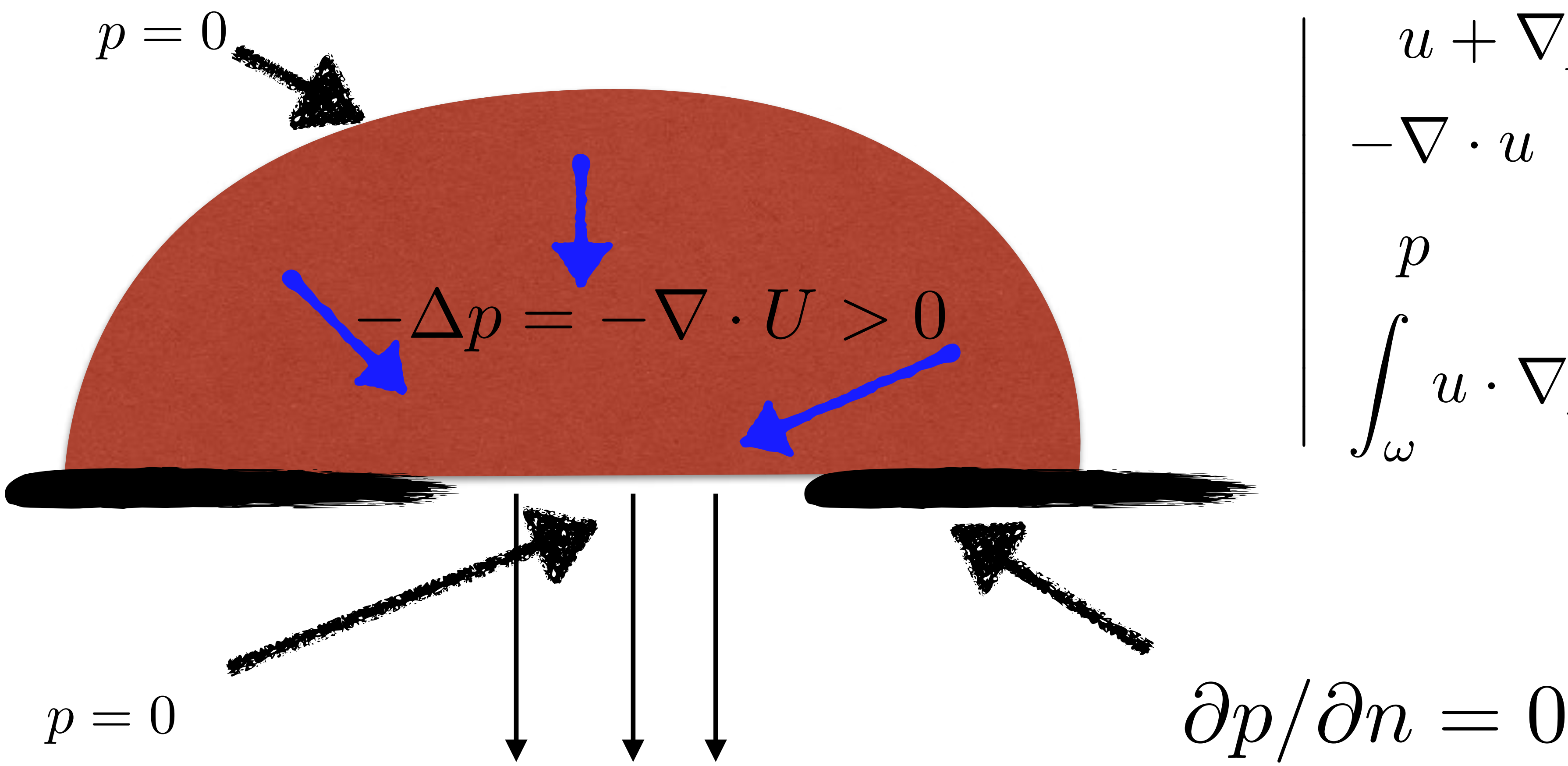
If $-\nabla \cdot U > 0$, the constraint is saturated, and one can eliminate the velocity, to obtain

$$-\Delta p = -\nabla \cdot U > 0$$

Evacuation of a room

$$u \quad = \quad P_{C_\rho} U$$





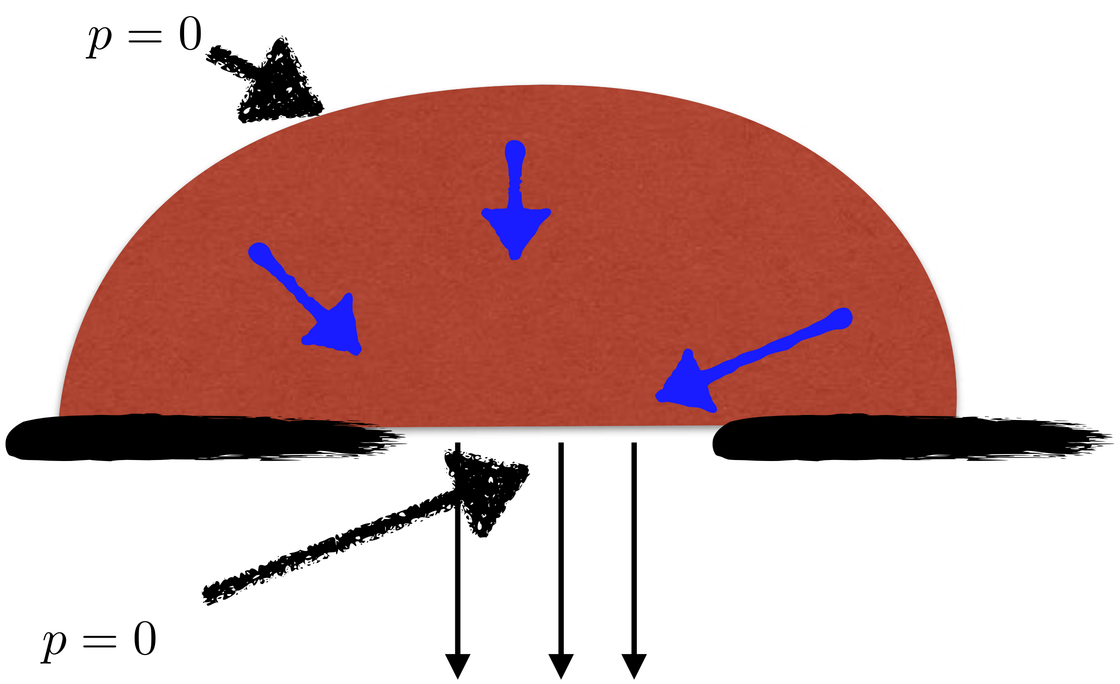
$$\left| \begin{array}{rcl} u + \nabla p & = & U \\ -\nabla \cdot u & \leq & 0 \\ p & \geq & 0 \\ \displaystyle\int_\omega u \cdot \nabla p & = & 0, \end{array} \right.$$

p>0 so that, at the exit, $u \cdot n = U \cdot n - \dfrac{\partial p}{\partial n} \geq U \cdot n$

People exit **faster** as they would if they were alone : **no capacity drop, no clogging, …**

Faster is slower effect ?

$\beta \approx 1$ correction factor



$$-\Delta p_\beta = -\nabla \cdot \beta U$$

$$J(\beta) = \int_{\Gamma_{out}} u_\beta \cdot n = \int_{\Gamma_{out}} U \cdot n - \int_{\Gamma_{out}} \frac{\partial p_\beta}{\partial n}$$

The gradient of J is $U \cdot \nabla q$

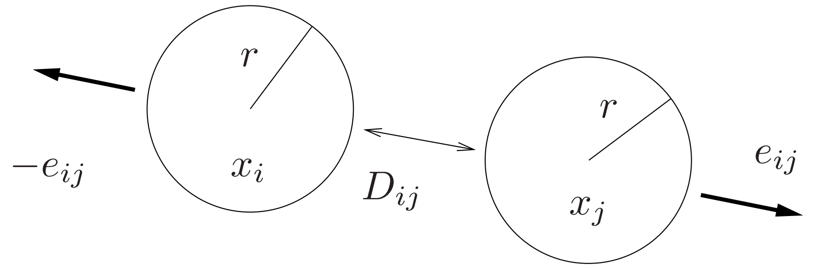
where q solves the adjoint problem :

$$\left| \begin{aligned} -\Delta q &= 0 && in\ \Omega, \\ q &= 1 && on\ \Gamma_{out} \\ q &= 0 && on\ \Gamma_{up} \\ \frac{\partial q}{\partial n} &= 0 && on\ \Gamma_w \end{aligned} \right.$$

∇q goes in the direction of U: Faster is Faster effect

Back to microscopic

$U = (U_1, \ldots, U_N)$ spontaneous velocities



Constraint $G_{ij} \cdot u \geq 0$ when i and j are in contact

$$G_{ij} = \nabla D_{ij}(x) = (0, \ldots, 0, -e_{ij}, 0, \ldots, 0, e_{ij}, 0, \ldots, 0) \in \mathbb{R}^{2N}$$

Micro

$$\left| \begin{array}{rcl} u - \sum_{i \sim j} p_{ij}\, G_{ij} & = & U, \\ -G_{ij} \cdot u & \leq & 0 \quad \forall i \sim j, \\ & p \geq 0, & \\ G_{ij} \cdot u > 0 \Longrightarrow p_{ij} = 0. & & \end{array} \right.$$

$$\left| \begin{array}{lcl} u + B^{\star} p & = & U, \\ Bu & \leq & 0, \\ p & \geq 0, & \\ p \cdot Bu & = & 0. \end{array} \right.$$

Macro

$$\left| \begin{array}{rcl} u + \nabla p & = & U \\ -\nabla \cdot u & \leq & 0 \\ p & \geq & 0 \\ \int_\omega u \cdot \nabla p & = & 0, \end{array} \right.$$

Discrete Poisson problem

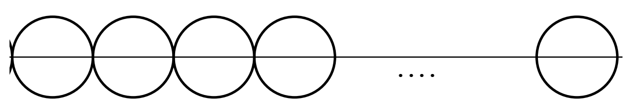
Discrete

$$BB^\star p = BU$$

Continuous

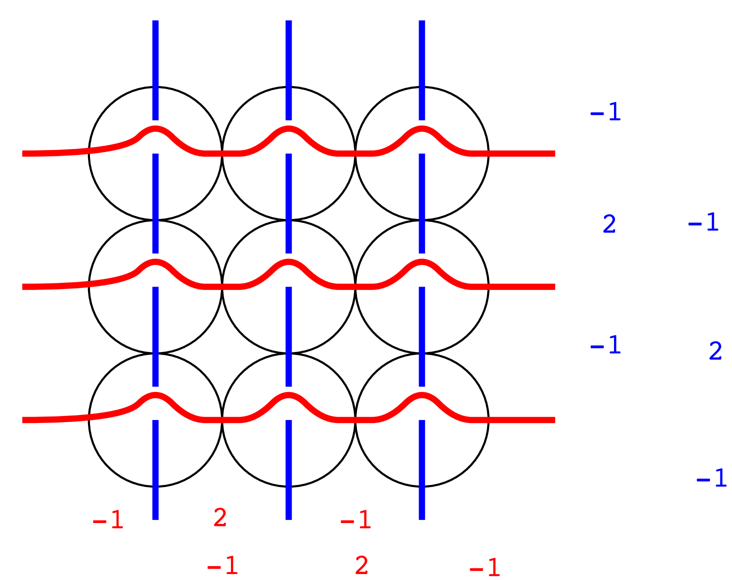
$$-\Delta p = -\nabla \cdot U > 0$$

In 1d :

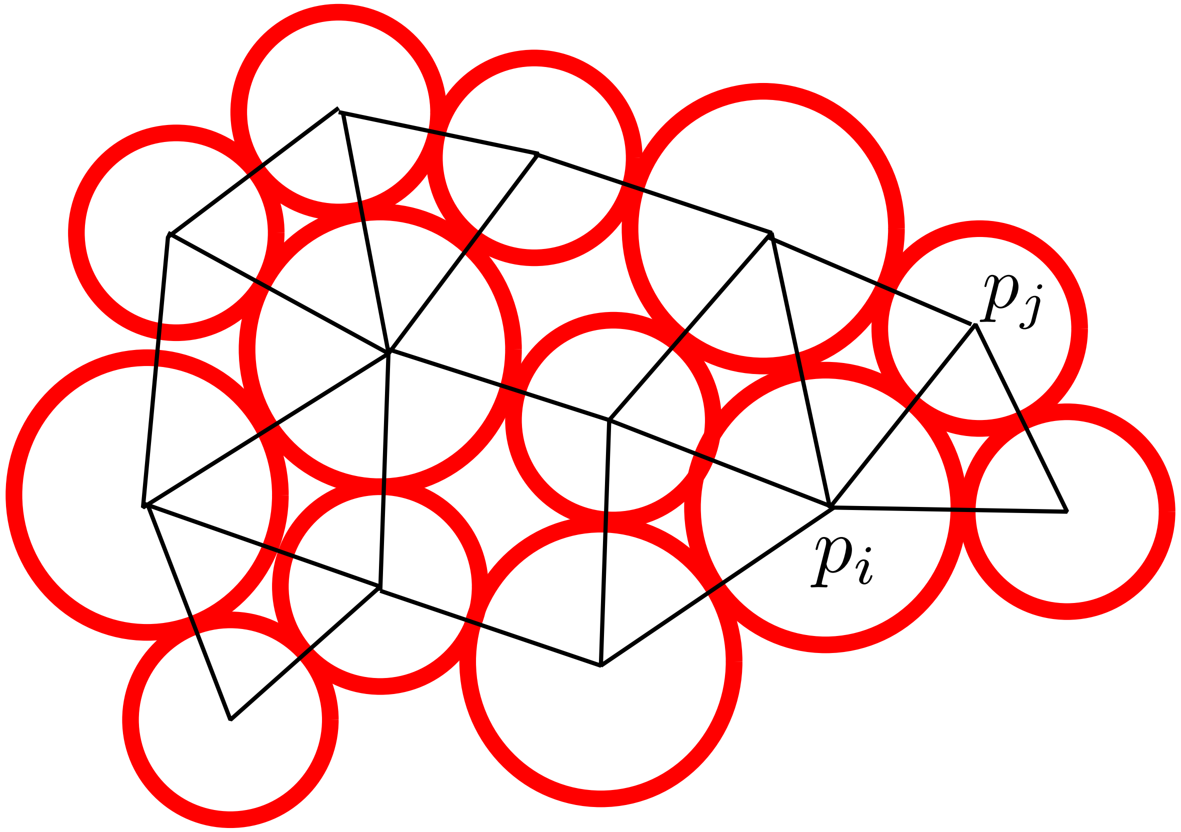


$$B = \begin{pmatrix} 1 & -1 & 0 & \cdot & \cdot & 0 \\ 0 & 1 & -1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & 1 & -1 \end{pmatrix} \qquad BB^\star = \begin{pmatrix} 2 & -1 & 0 & \cdot & \cdot & 0 \\ -1 & 2 & -1 & 0 & \cdot & \cdot \\ 0 & -1 & \cdot & \cdot & & \cdot \\ \cdot & & \cdot & \cdot & \cdot & \cdot \\ \cdot & & & \cdot & 2 & -1 \\ 0 & \cdot & \cdot & 0 & -1 & 2 \end{pmatrix}$$

In higher dimensions : a bit different



Remark : « Standard » laplacian on the **primal** network



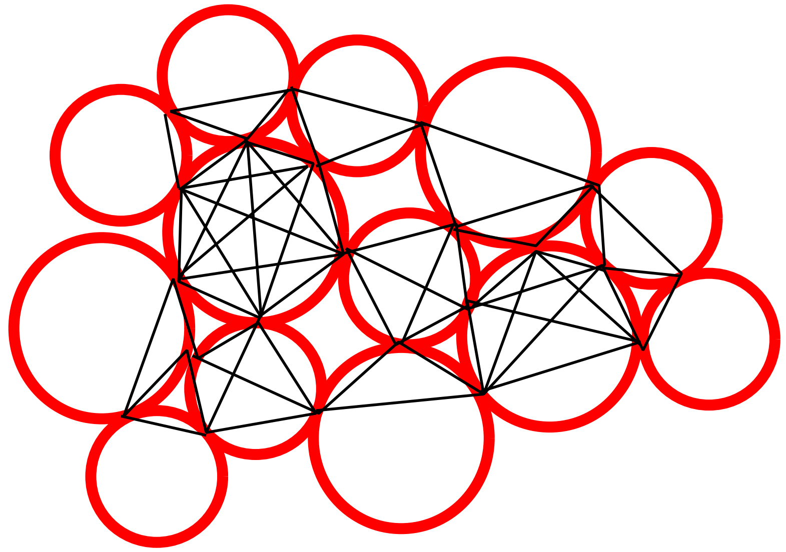
Macroscopic

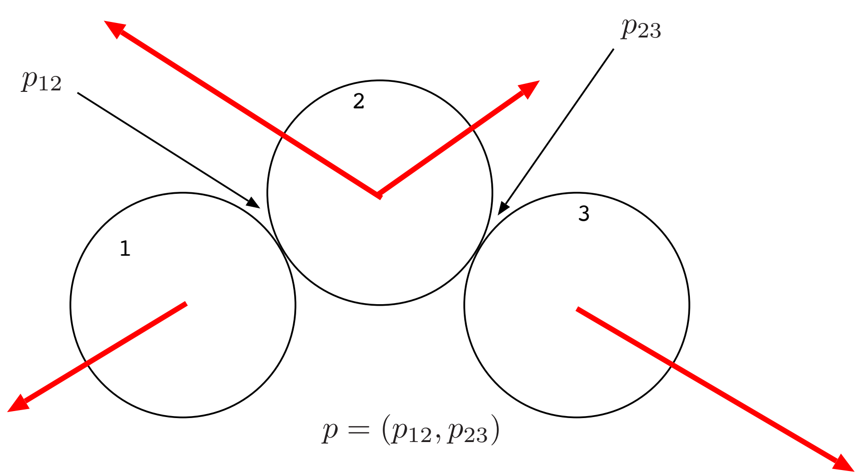
Ohm / Poiseuille / Fick law	$u_{ij} = -c_{ij}(p_j - p_i)$	$u = -k\nabla p$
Kirchhoff law	$\sum_{j\sim i} u_{ij} = 0$	$\nabla \cdot u = 0$
Discrete harmonicity	$\sum_{j\sim i} c_{ij}(p_i - p_j) = 0$	$-\nabla \cdot k\nabla p = 0$

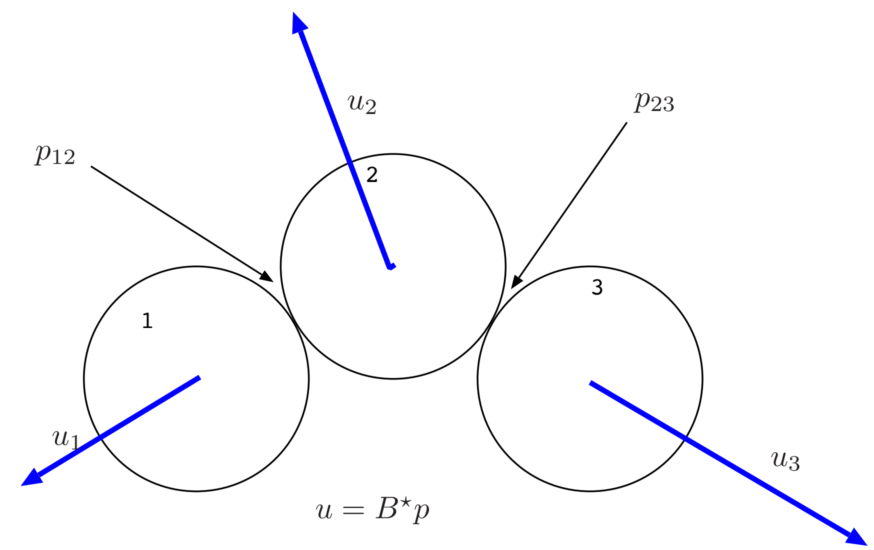
Maximum principle

Here : non-standard laplacian on the **dual** network

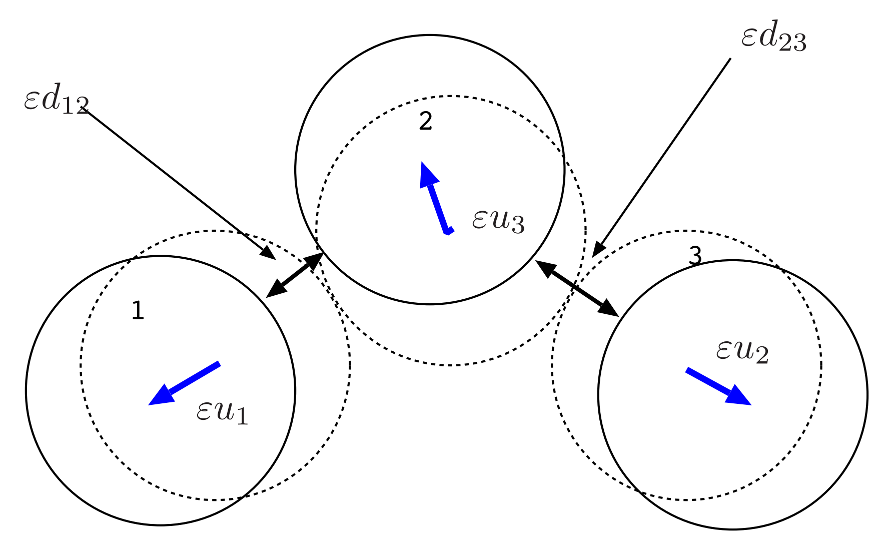
Pressures defined at **edges** (i.e. contact points)

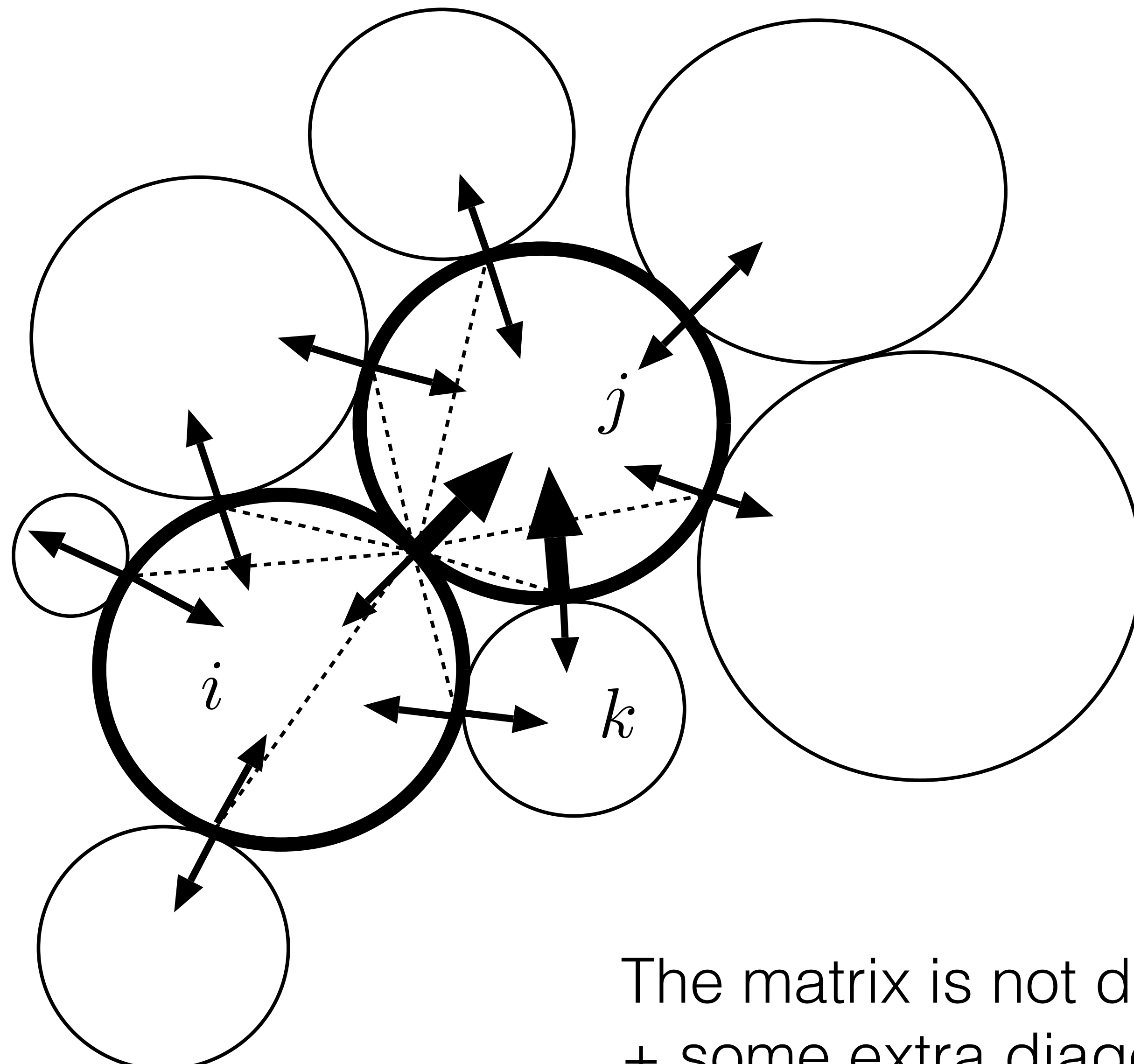






$$p \longmapsto BB^\star p$$

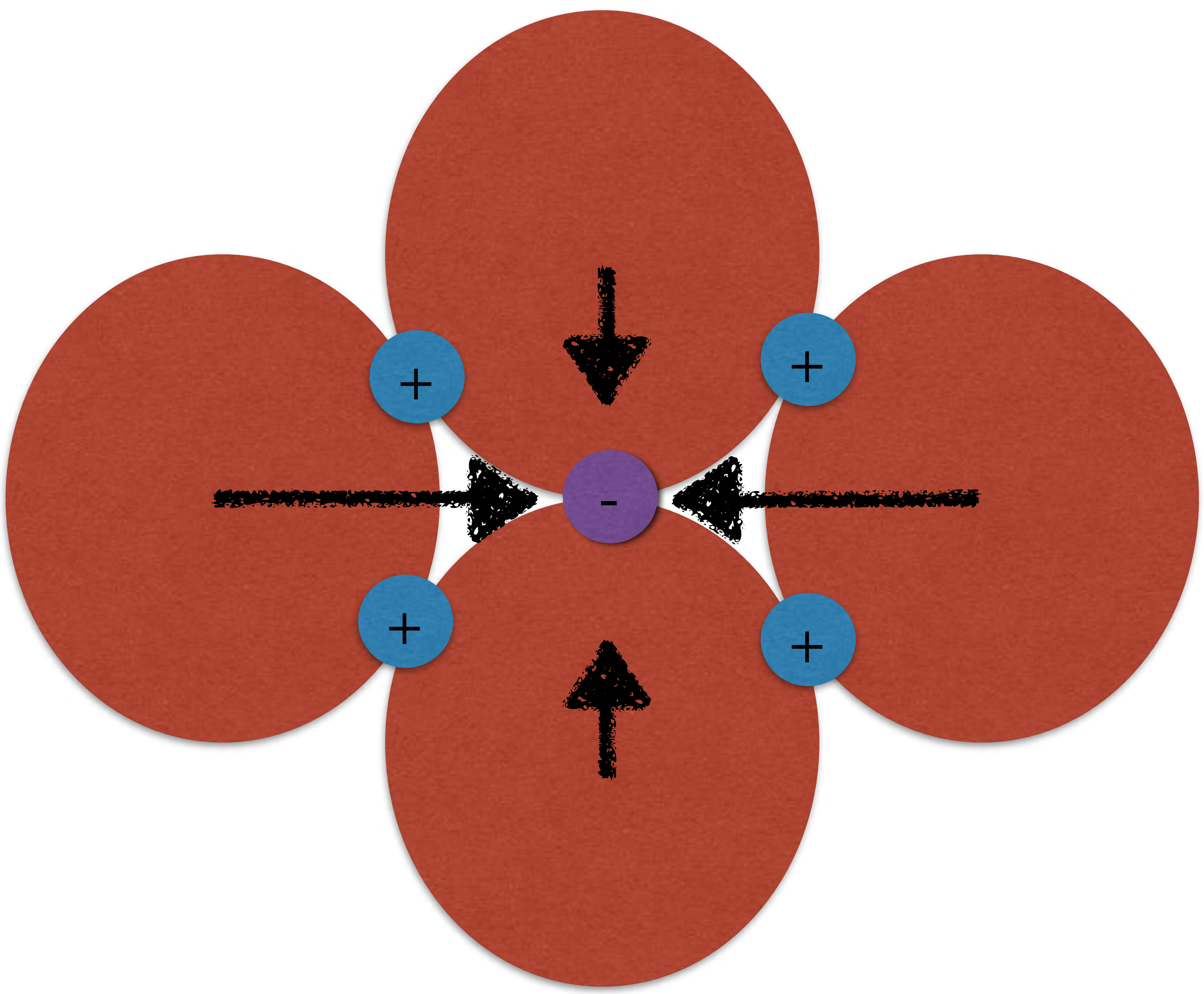




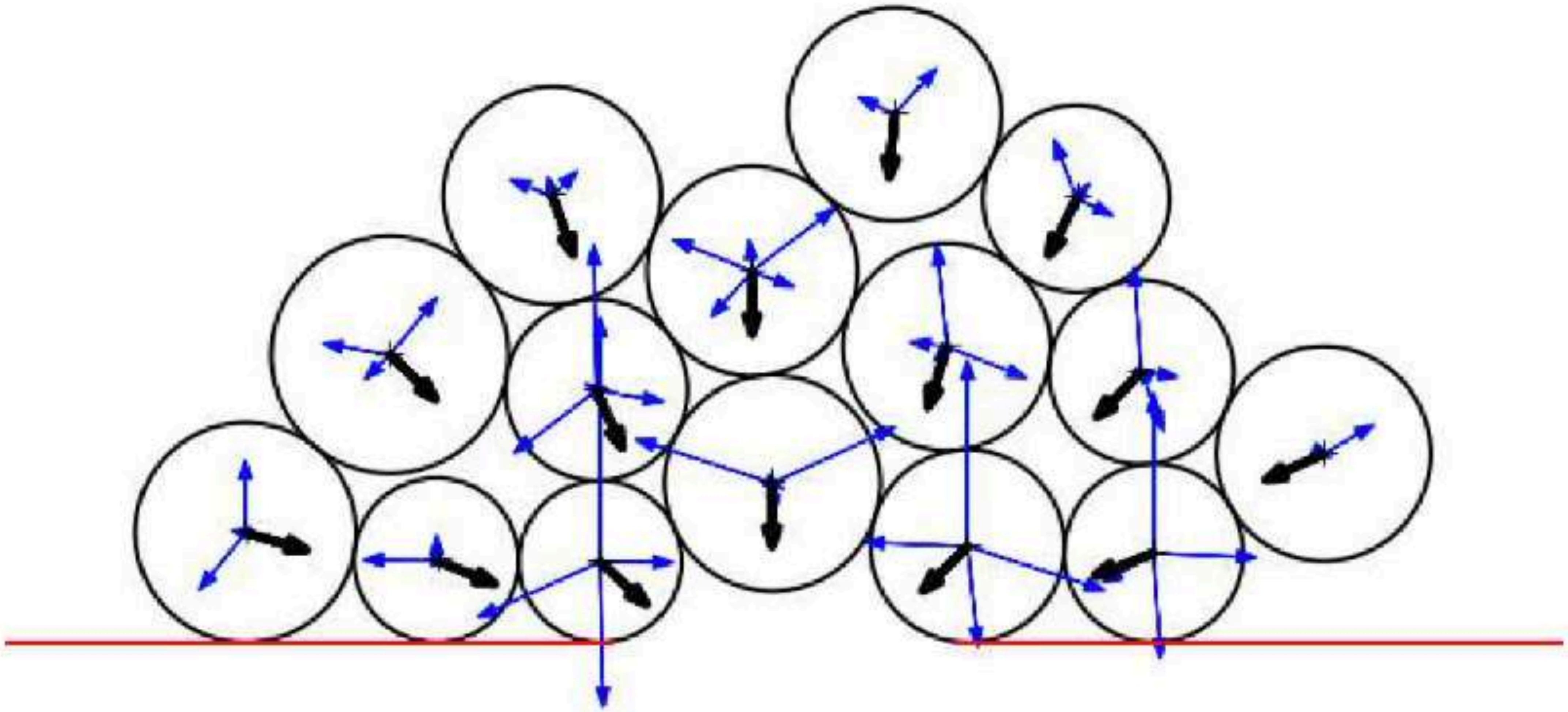
The matrix is not diagonally dominant in general
+ some extra diagonal terms are **positive**

Consequence : no maximum principle

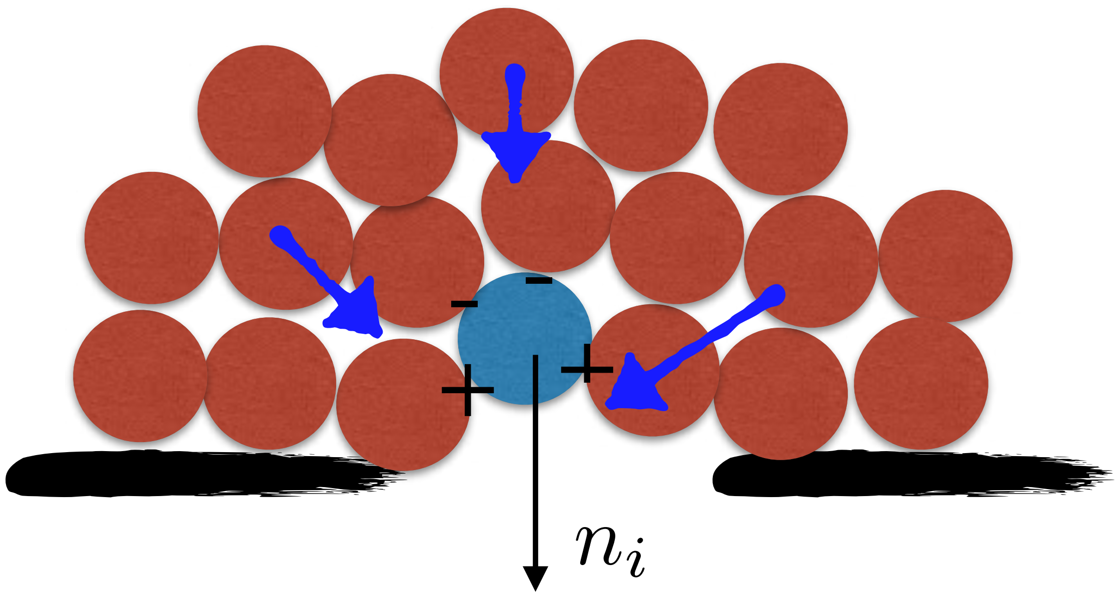
$BB^{\star}p = BU > 0$ does **not** imply $p > 0$



Static jams



Faster is slower effect in the microscopic situation ?



$\beta = (\beta_i)_i$ speed correction factors

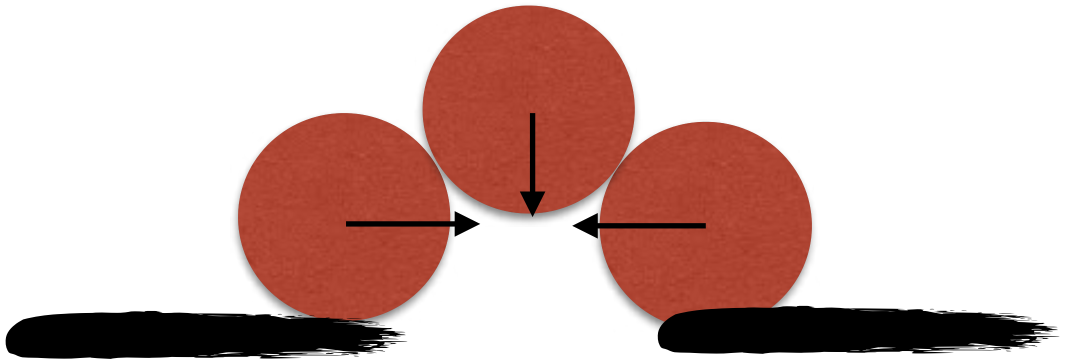
$$BB^\star p_\beta = B(\beta \odot U)$$

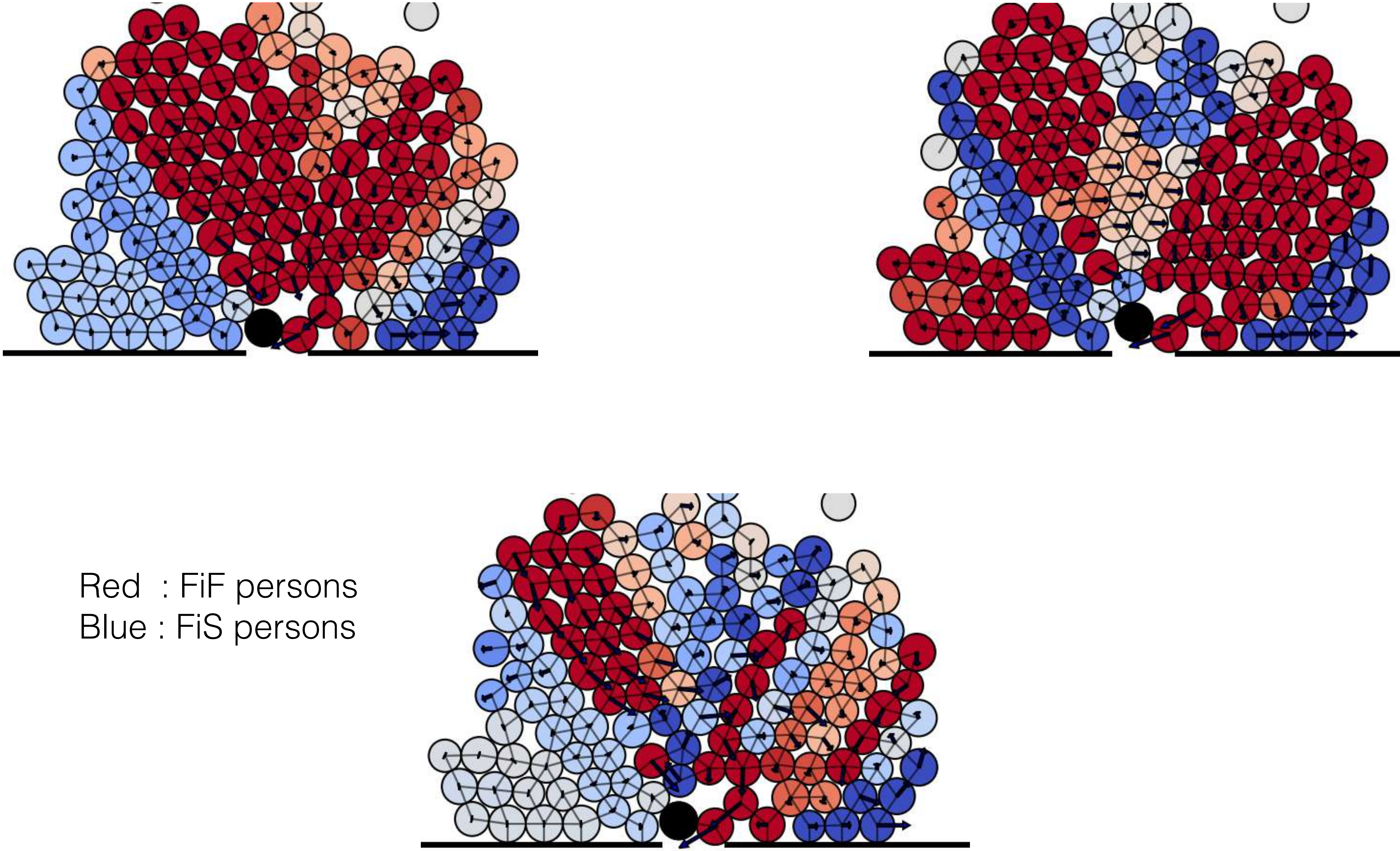
$$J(\beta) = -B^\star p \cdot n_i$$

The gradient of J is $U \odot B^\star q$

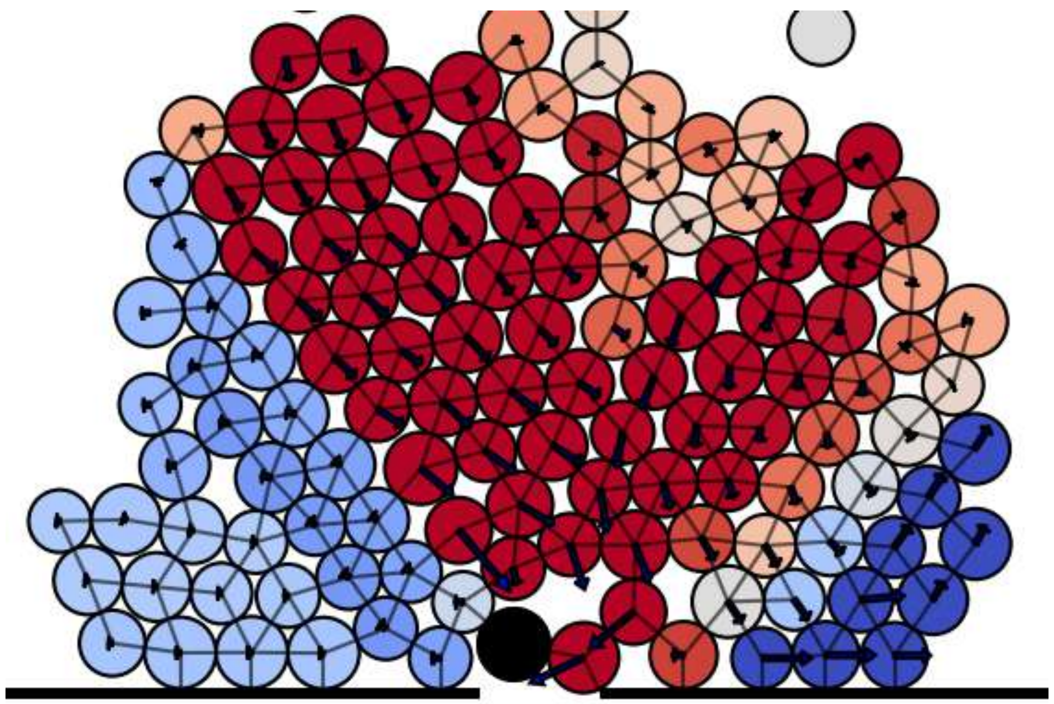
where q solves the adjoint problem : $BB^\star q = Bn_i$

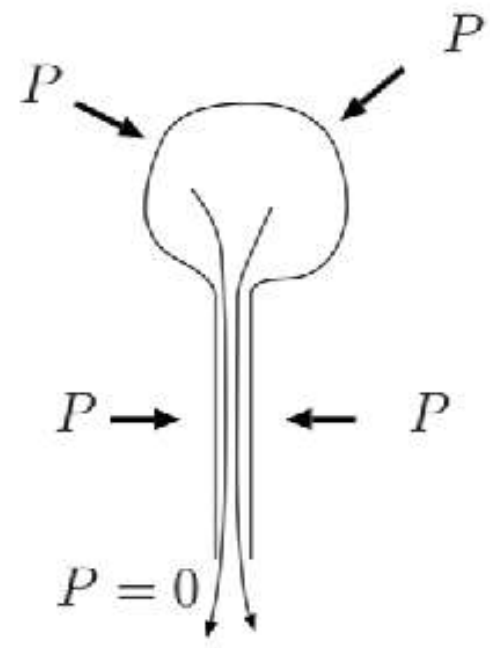
No reason for J to be positive: some individuals may accelerate the egress of the blue guy by **reducing** their speed

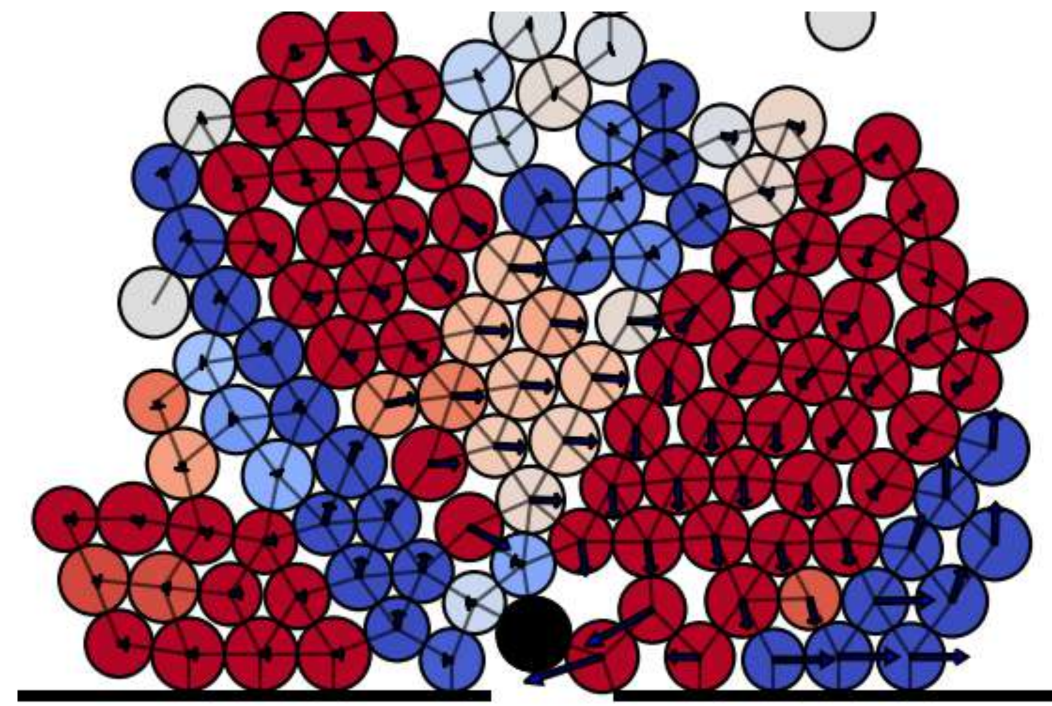




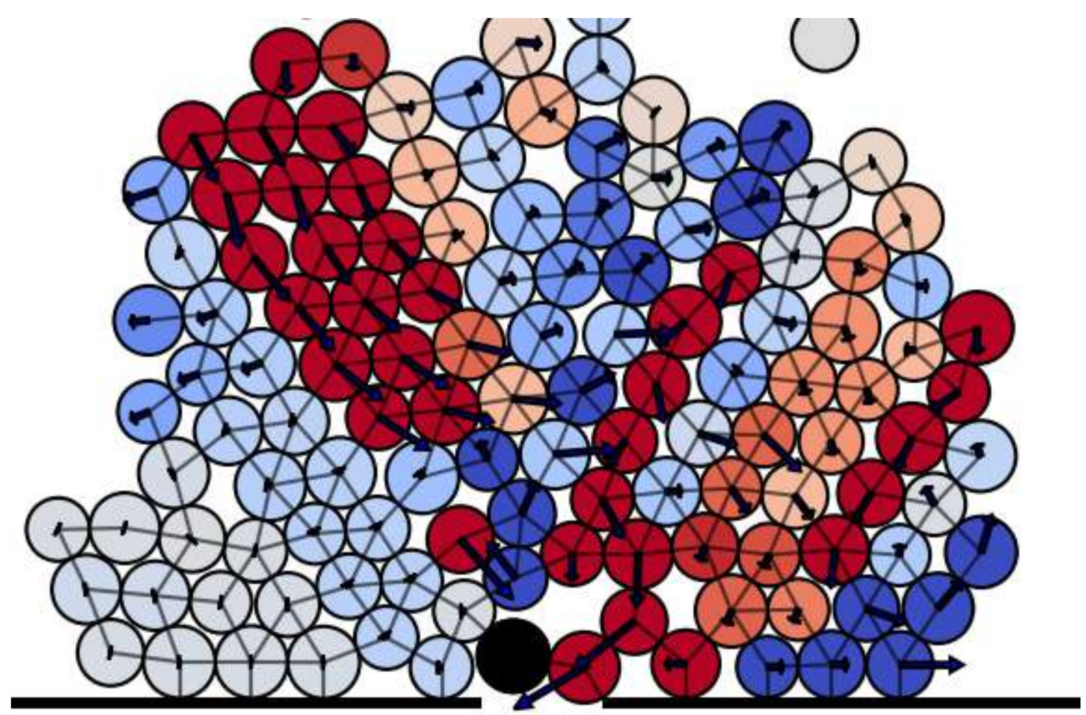
Red : FiF persons
Blue : FiS persons







Red : FiF persons
Blue : FiS persons



To Sum up

Parameter-less hard-sphere model makes it possible to reproduce FiS effect
« Mathematical » explanation : bad properties of an underlying discrete Laplacian

Model is frictionless at microscopic model, but the rigid grain approach my be considered to model some sort of mesoscopic friction