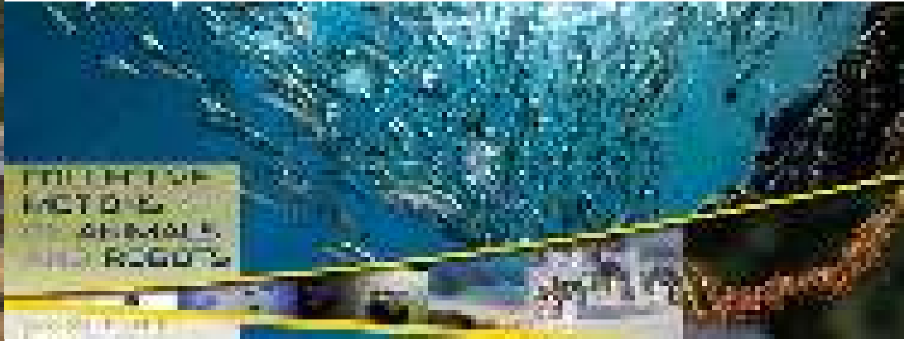

How to model the dynamics of pedestrians?

Walking a fine line between game theory,
physics and transportation science

28 mai 2024

Alexandre NICOLAS

- **Iñaki ECHEVERRÍA-HUARTE** (Univ. Navarra)
- **Denis ULLMO, Matteo BUTANO, Cécile APPERT-ROLLAND, Thibault BONNEMAIN** (Univ. Paris-Saclay)
- **Jakob CORDES** (Jülich Forschungszentrum)
- **Oscar DUFOUR, Alexis RAULIN FOISSAC, Antonin ROGE** (Univ. Lyon 1)

Crowd dynamics

X

Pastor et al., *Phys. Rev. E* (2015)

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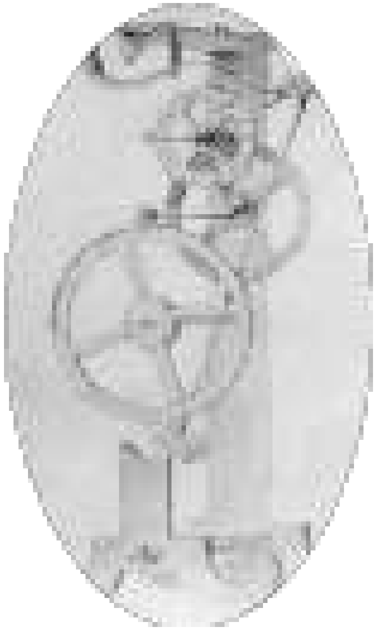
Where mechanics *meets* decision-making

A (biased) historical perspective

Beginning of XVIIth century

Galileo Galilei

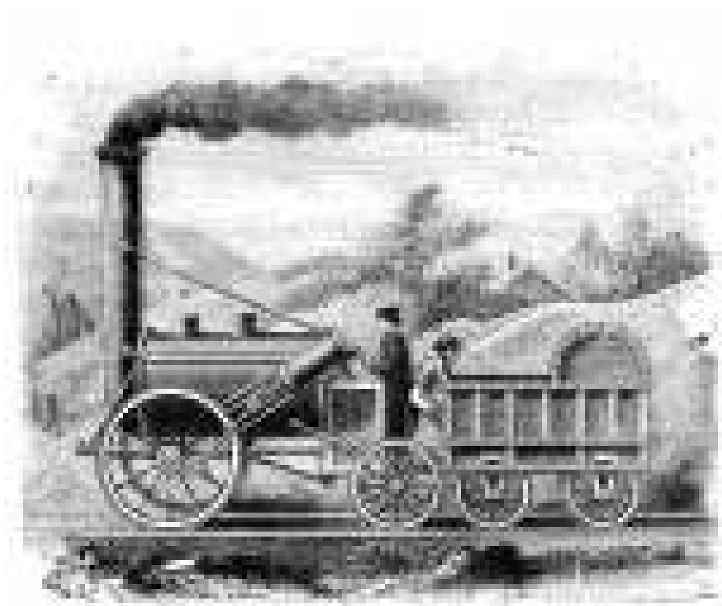
(1564-1642)



$$\ddot{x} = 0$$

End of the XIXth century

Ludwig Boltzmann
(1844-1906)



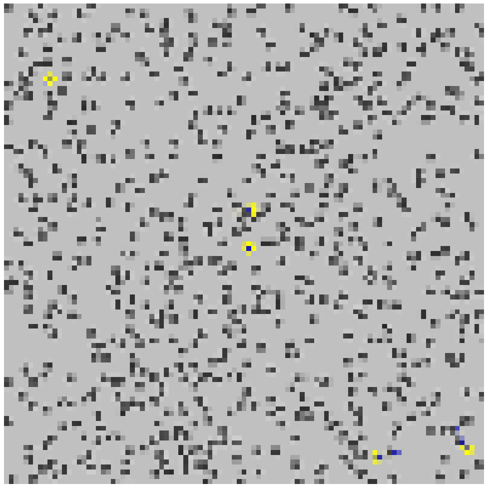
Steam engine

Mechanistic foundation
of the entropy of a gas

Newton's equation of motion

$$m\ddot{\boldsymbol{r}} = \boldsymbol{F}$$

Beginning of XXth century



Stochastic motion induces diffusion in suspensions

Paul Langevin
(1872-1946)

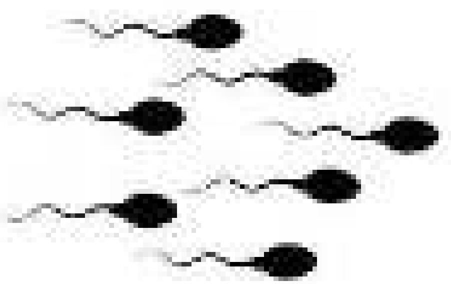
Albert Einstein
(1879-1955)

Robert Brown
(1773-1858)

Langevin equation

$$m\ddot{\mathbf{r}}_k + \gamma\dot{\mathbf{r}}_k = \boldsymbol{F} + \text{noise}$$

End of XXth century

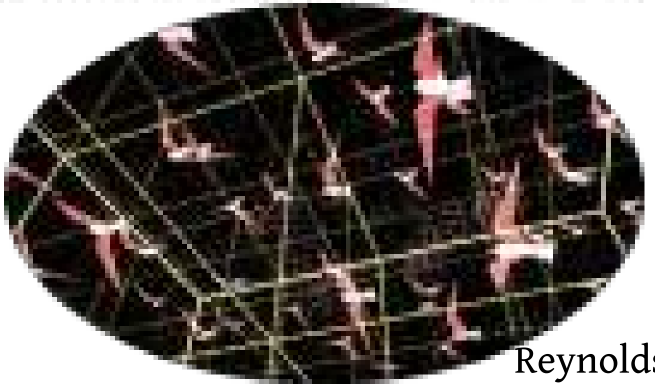


Active matter

Self-propelled particle

$$m\ddot{\vec{r}}_k + \gamma\dot{\vec{r}}_k = \frac{m}{\tau} v_0 - \sum_k F_{i\to j} + \text{noise}$$

End of XXth century



Reynolds' boid model

Tamás Vicsek
(1948 - ...)

Craig Reynolds
(1953 - ...)

Modified interaction forces between agents

- Alignment force acting on « birds' » orientations

  Vicsek et al., *Phys. Rev. Lett.* (1995)
  Toner & Tu, *Phys. Rev. Lett.* (1995)

- Repulsive « social forces » between pedestrians

  Helbing & Molnar, *Phys. Rev. E* (1995)

- Asymmetric perception

  Lavergne et al., *Science* (2019)

$$m\dot{v}_i + \gamma v_i = \frac{m}{\tau} v_i^0 + \sum_{\text{perceived } j} \vec{F}_{i\to j} + \text{noise}$$

Handling interactions in this way may be insufficient

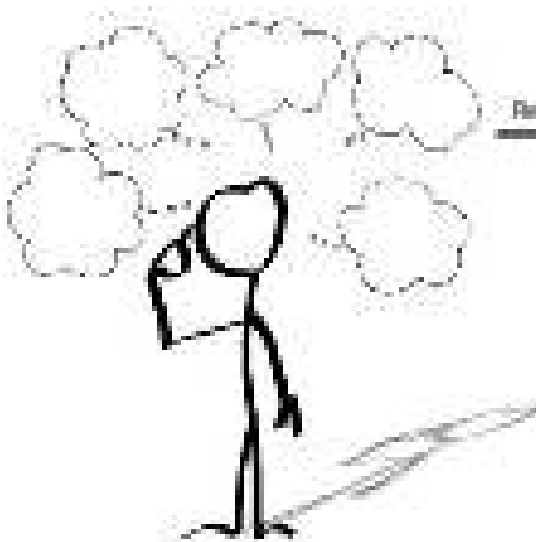
Mobility in the Metaverse

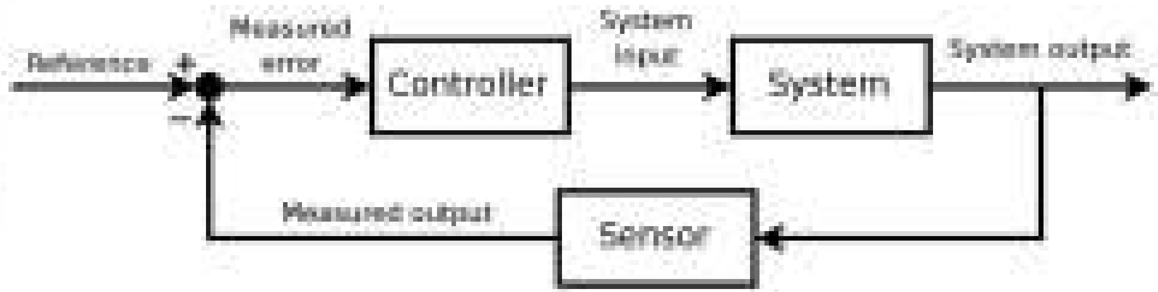
Robot swarm

Conway's game of life

https://arxiv.org/abs/2404.03071

Handling interactions in this way may be insufficient





Claim : For **pedestrians** (and **robots**) (and **animals**?), the propulsion term v_0 results from a ***decision*** that ***anticipates*** the motion of the other agents.

⇒ Need to borrow concepts from other fields

Also see Bovy & Hoogendoorn, *Optimal Control...* (2003)
Van Toll, ..., Pettré, *Symposium on Interactive 3D...* (2020)

I. A (biased) historical perspective

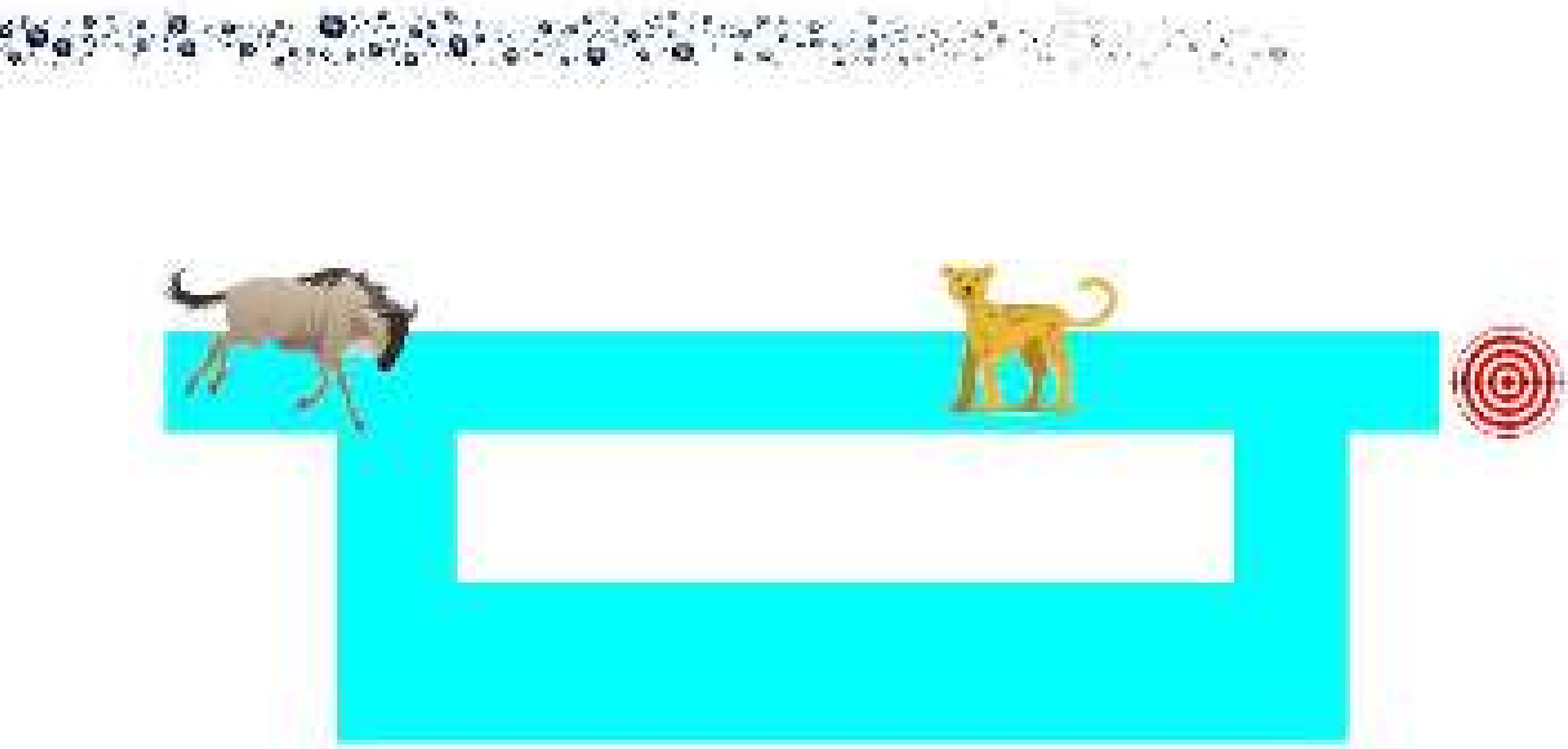
II. Some evidence of decision-making in local navigation

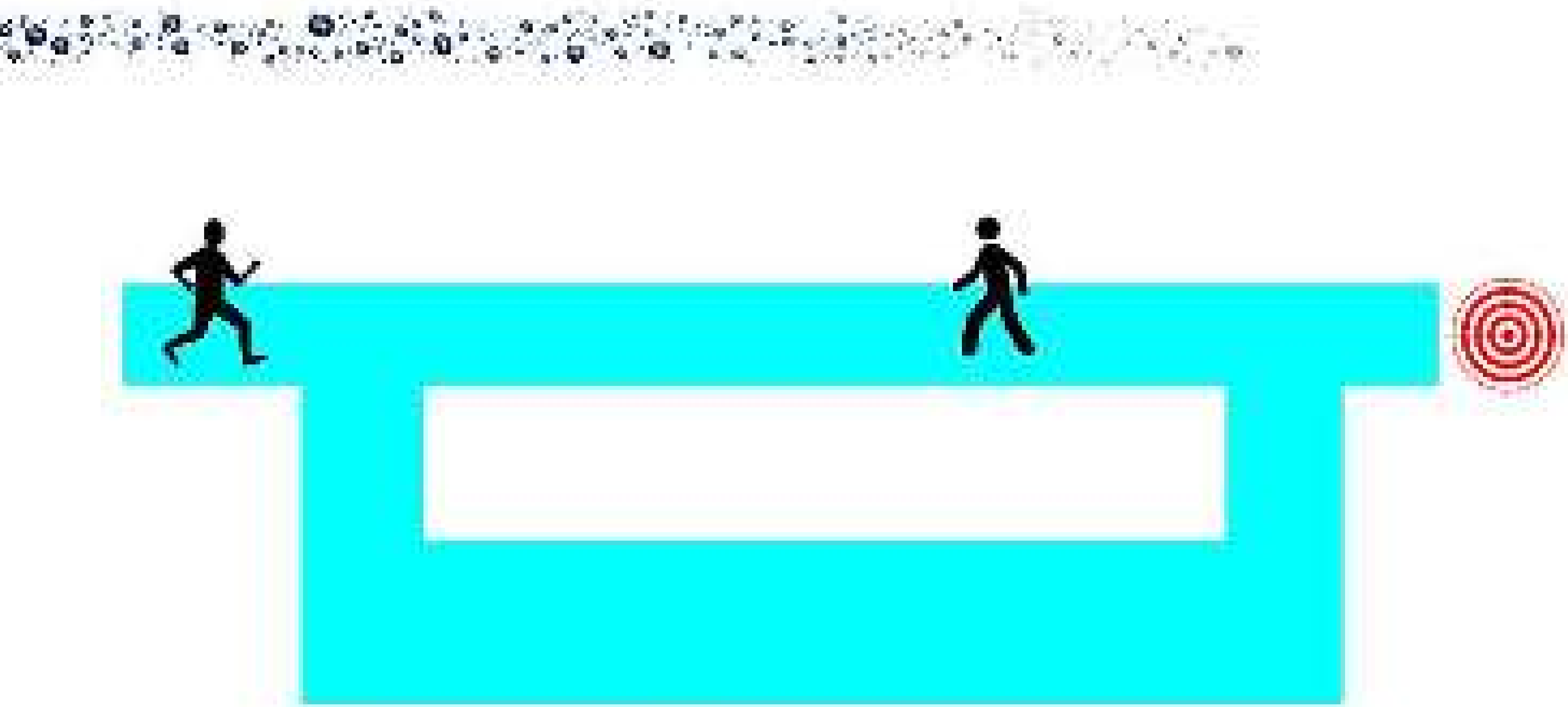
III. Basics of optimal control and game theory

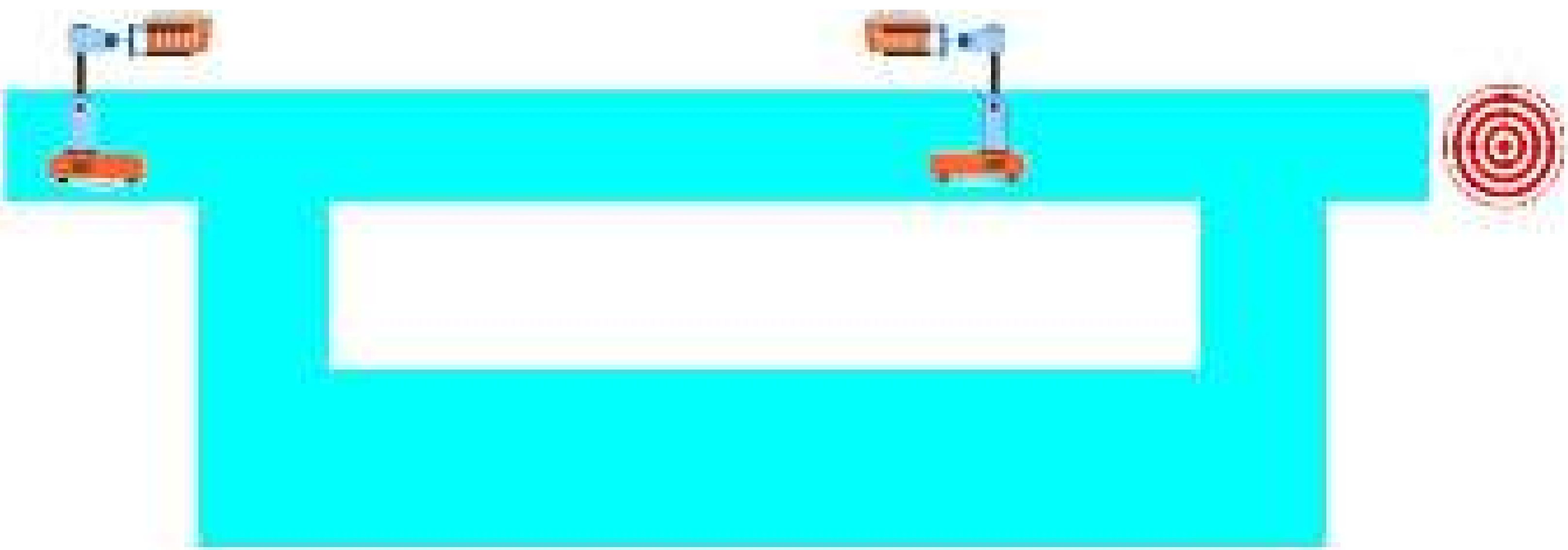
IV. Application to pedestrian crowds

Some evidence of anticipation and decision-making

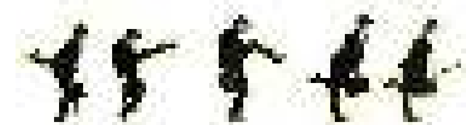

BBC one







Different levels of descriptions



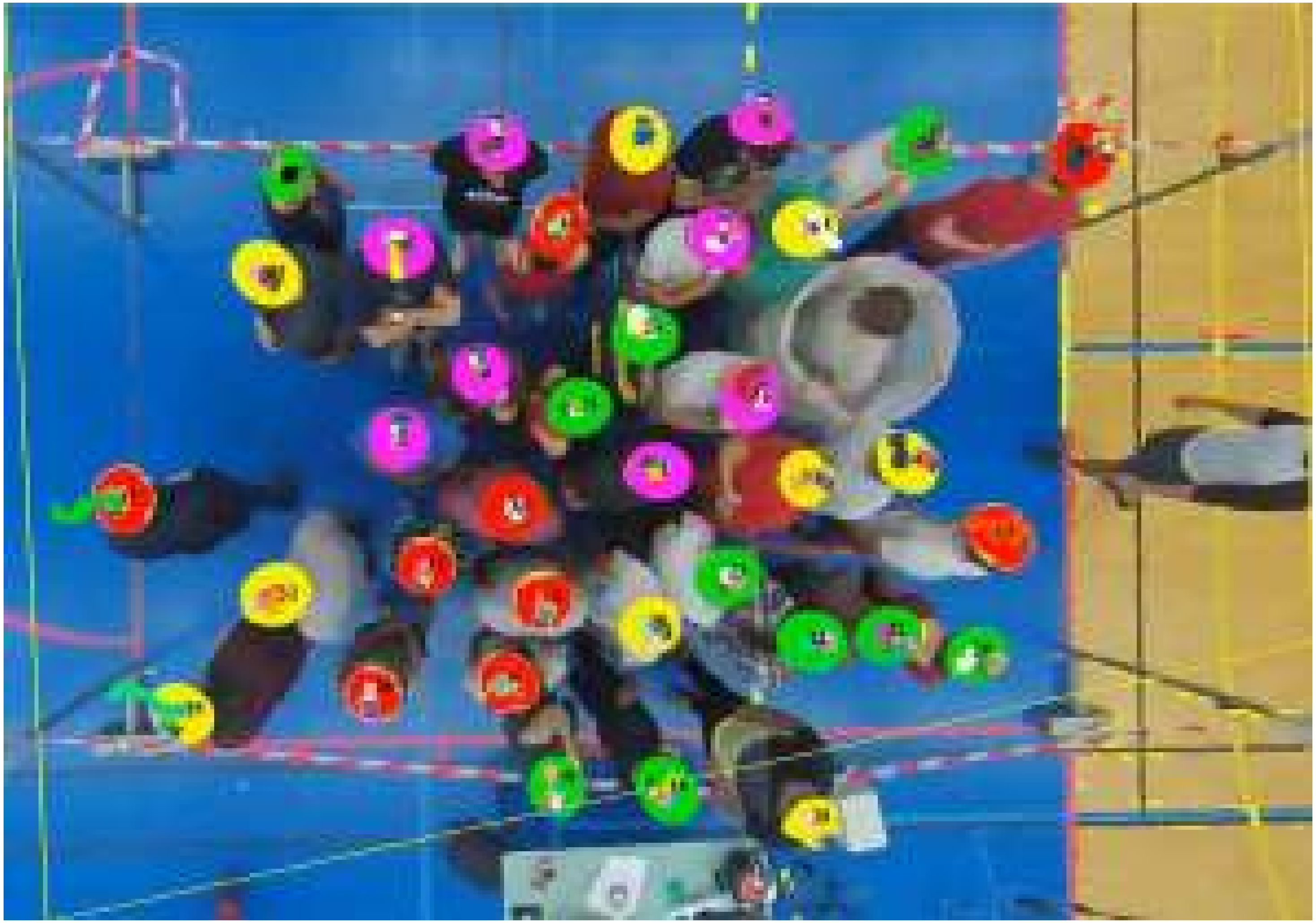
where and when you want to go

which route/path and schedule you follow to get there

how you walk

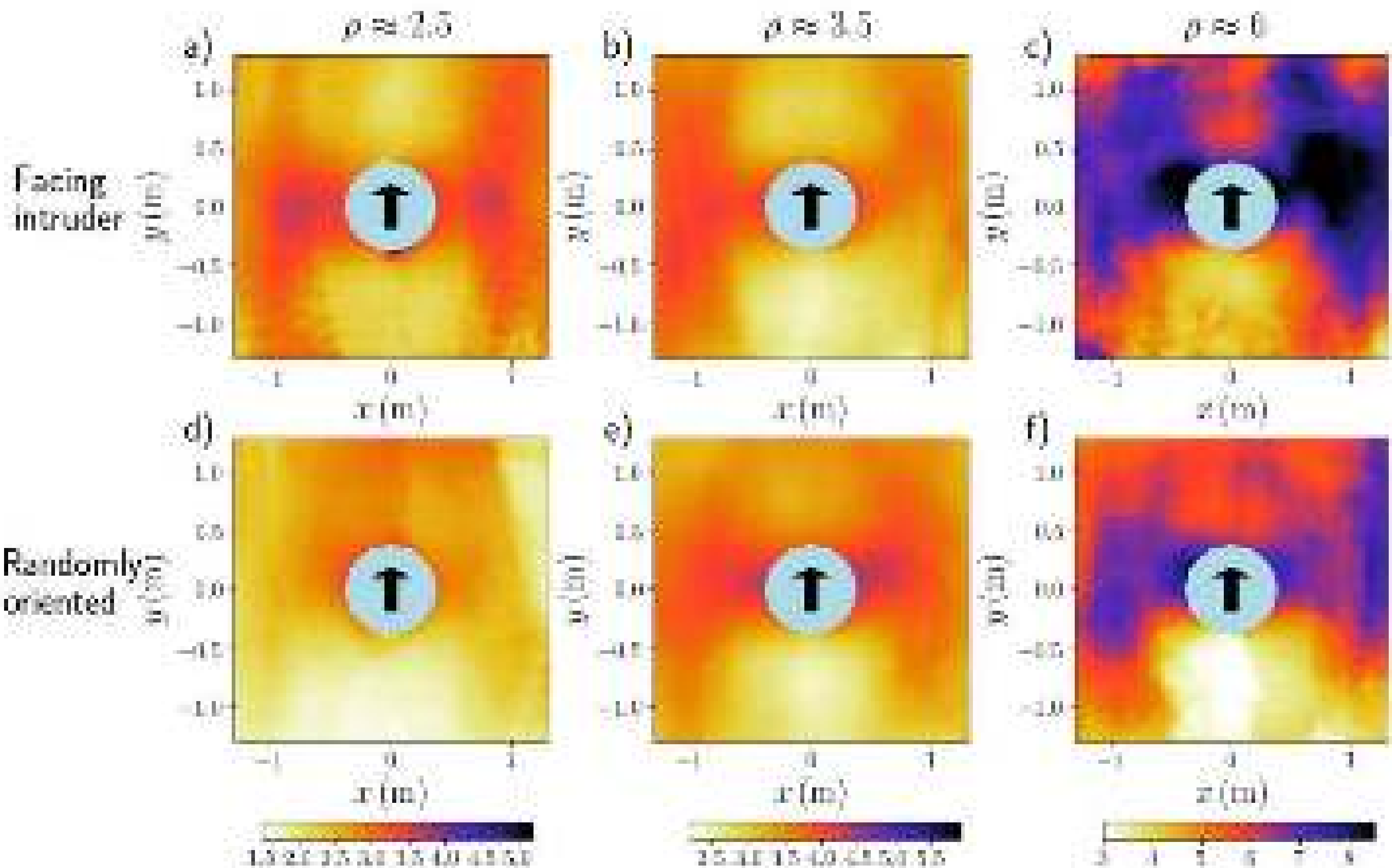
Hoogendoorn (2001)
Hoogendoorn & Bovy, *TRB* (2002)

Crossing of a static crowd



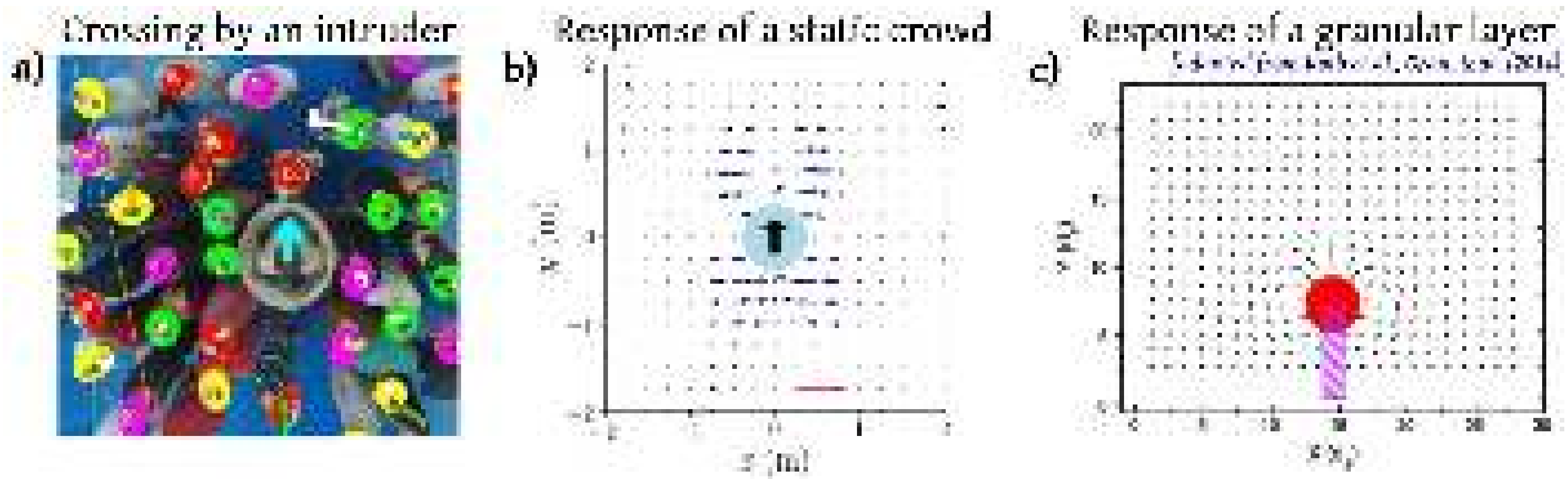
About 40 participants, various orientations, various densities, etc.

Crossing of a static crowd



Density field around the intruder

Crossing of a static crowd



Displacement field, in the intruder’s co-moving frame

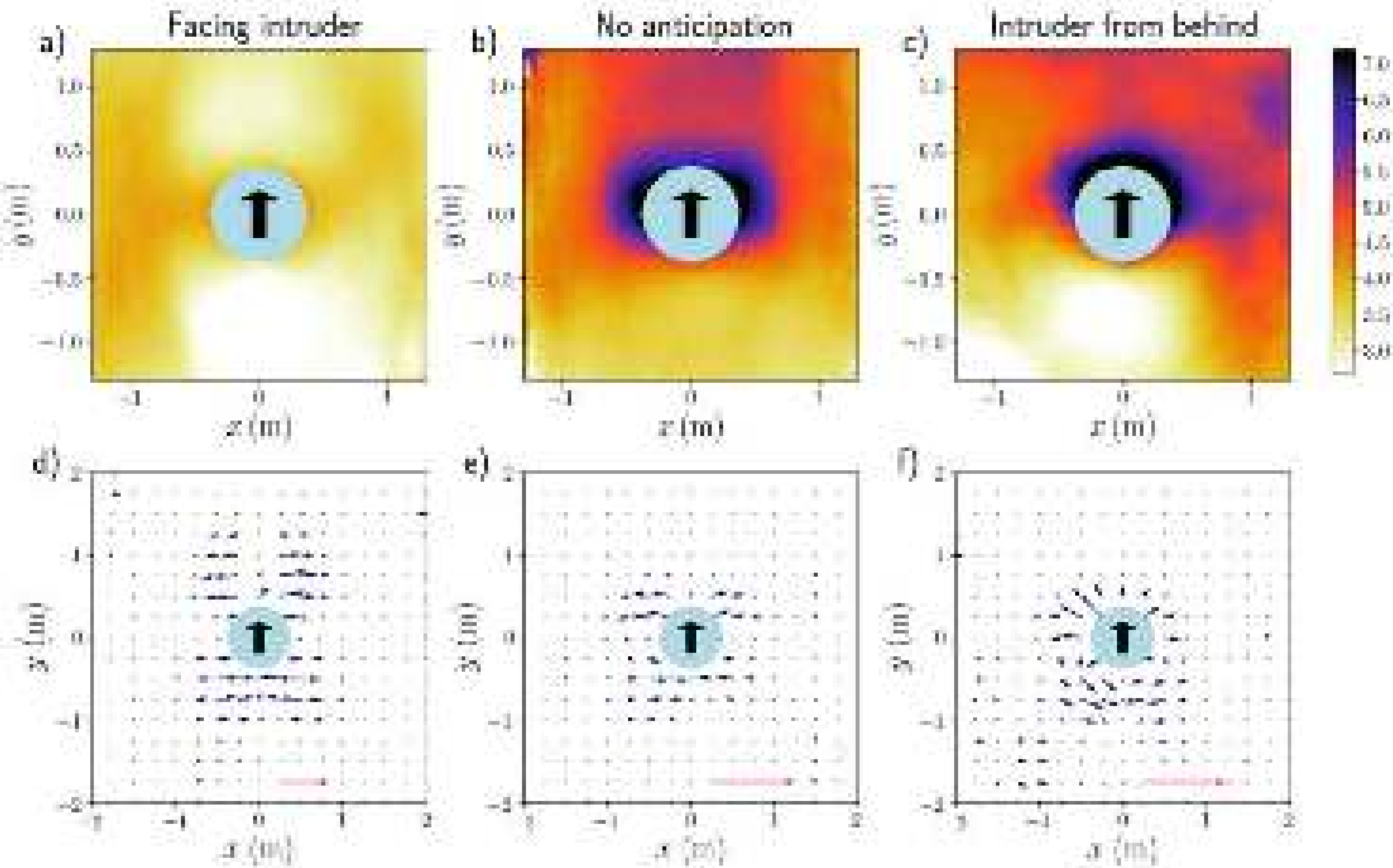
- Anticipation
- Self-propulsion + avoidance *strategy*

<u>Nicolas</u>, Bouzat, Ibañez, Kuperman & Appert-Rolland, *Sci. Rep.* (2019)
Also see Métivet, Pastorello & Peyla, *EPL* (2018)

Crossing of a static crowd

Density field

Velocity field



Nicolas, Bouzat, Ibañez, Kuperman, Appert-Rolland, *Sci. Rep.* (2019)

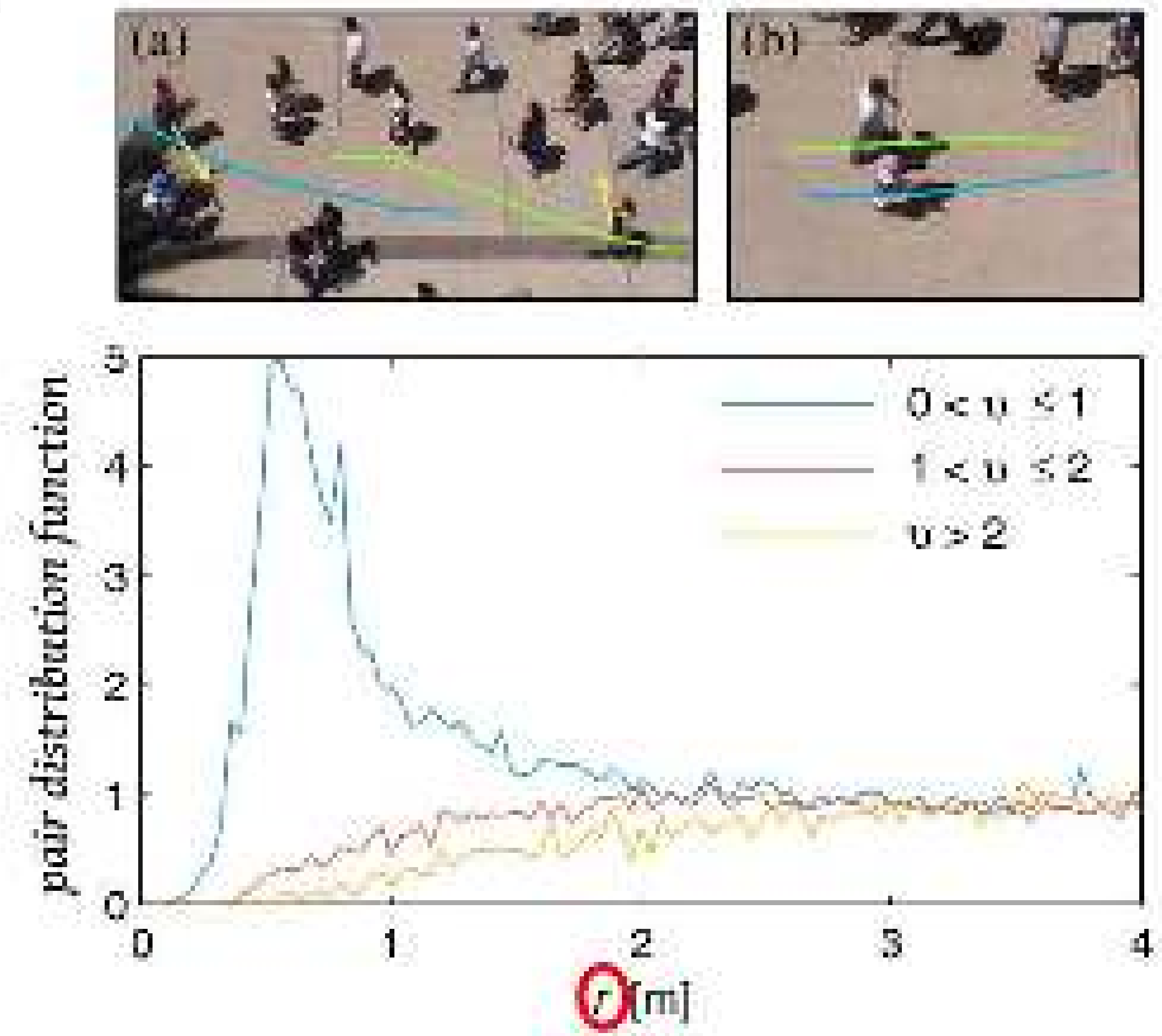
Also see Métivet, Pastorello & Peyla, *EPL* (2018)

Crossing of a static crowd

2019-2020 demonstrations in Hong-Kong

(© A. Chow)

Empirical data on pedestrian spacings

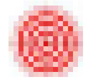


Radial distribution function

Karamouzas, Skinner, Guy, *Phys. Rev. Lett.* (2014)

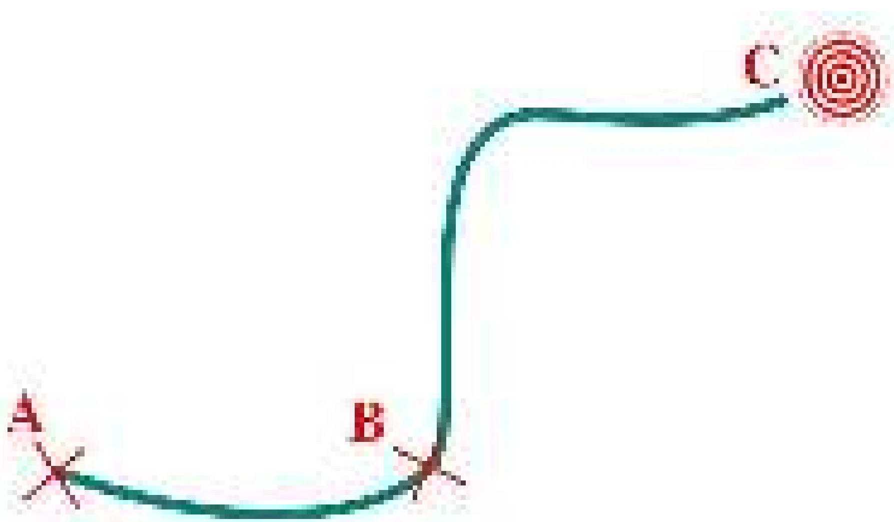
Optimal control theory

Problem setting

Your goal : find the path to  that minimises the cost C

$$C\left(\{\boldsymbol{r}\}_{t=0}^{T}\right) = \int_0^T c(\boldsymbol{r}_\tau, \boldsymbol{v}_\tau, \tau)\,d\tau + C_T\left(\boldsymbol{r}_T\right)$$

$$\dot{\boldsymbol{r}} = \boldsymbol{v}$$



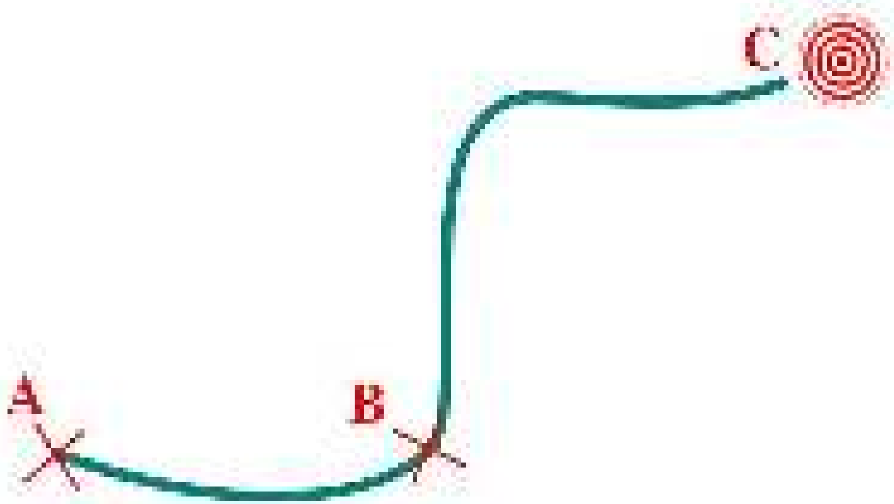
Problem setting

Your goal : find the path to that minimises the cost C

$$C\left(\{\boldsymbol{r}\}_{t=0}^{T}\right) = \int_0^T c(\boldsymbol{r}_\tau, \boldsymbol{v}_\tau, \tau)d\tau + C_T\left(\boldsymbol{r}_T\right)$$

$$\dot{\boldsymbol{r}} = \boldsymbol{v}$$

- Bellman's principle of optimality



An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.

The value function

- Assess the remaining cost

$$u(\mathbf{r},t) = \inf_{\mathbf{v}\in\mathbb{R}^x} \int_t^T c + C_T\left(\mathbf{r}_T\right)$$

$$= \int_t^T \underbrace{c(\mathbf{r}^*_\tau, \mathbf{v}^*_\tau, \tau)}_{c^*_\tau} d\tau + C_T\left(\mathbf{r}^*_T\right)$$

u is known at time T

The value function

- Continuation of optimality

$$
\begin{aligned}
u(\boldsymbol{r}, t+dt) &= \inf_{\mathbf{v}} \left\{ \int_t^{t+dt} c + \int_{t+dt}^{T} c + C_T(\boldsymbol{r}_T) \right\} \\
&= c_t^* dt + u(\boldsymbol{r}^*_{t-dt}, t+dt) \\
&= c_t^* dt + u(\boldsymbol{r}, t) - dt \frac{\partial u}{\partial t} + \boldsymbol{v}_t^* dt \cdot \nabla u
\end{aligned}
$$

where $\dfrac{\partial c_t}{\partial \boldsymbol{v}}(\boldsymbol{r}_t, \boldsymbol{v}_t^*, t) = -\nabla u$

- Hamilton-Jacobi-Bellman equation

$$
\boxed{0 = c_t^* + \frac{\partial u}{\partial t} + \boldsymbol{v}_t^* \cdot \nabla u}
$$

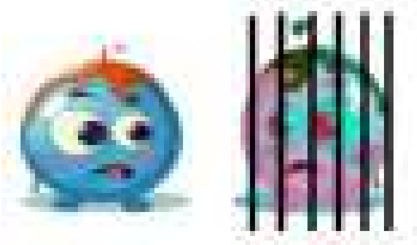

Game theory

A game between players

- *What if there are several players ?*
- Need to heed-predict-anticipate what the other(s) will do

Von Neumann & Morgenstern, *Theory of Games and Economic Behavior* (1944)

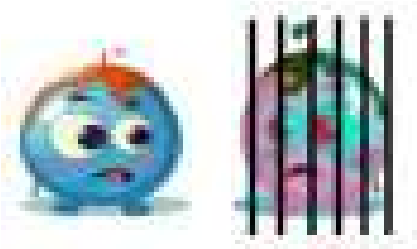
- Prisoners' dilemma (Tucker, 1950)



	Cooperate	Defect
Cooperate	1, 1	10, 0
Defect	0, 10	5, 5

Individual costs c (negative payoffs)

- Prisoners’ dilemma (Tucker, 1950)

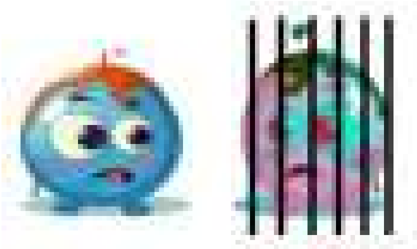


	Cooperate	Defect
Cooperate	1, 1	10, 0
Defect	0, 10	5, 5

Individual costs c (negative payoffs)

  - Pure strategy $s_j \in \{\mathrm{C}, \mathrm{D}\}$
  - Mixed strategy $s_j = p \cdot \mathrm{C} + (1 - p) \cdot \mathrm{D}$

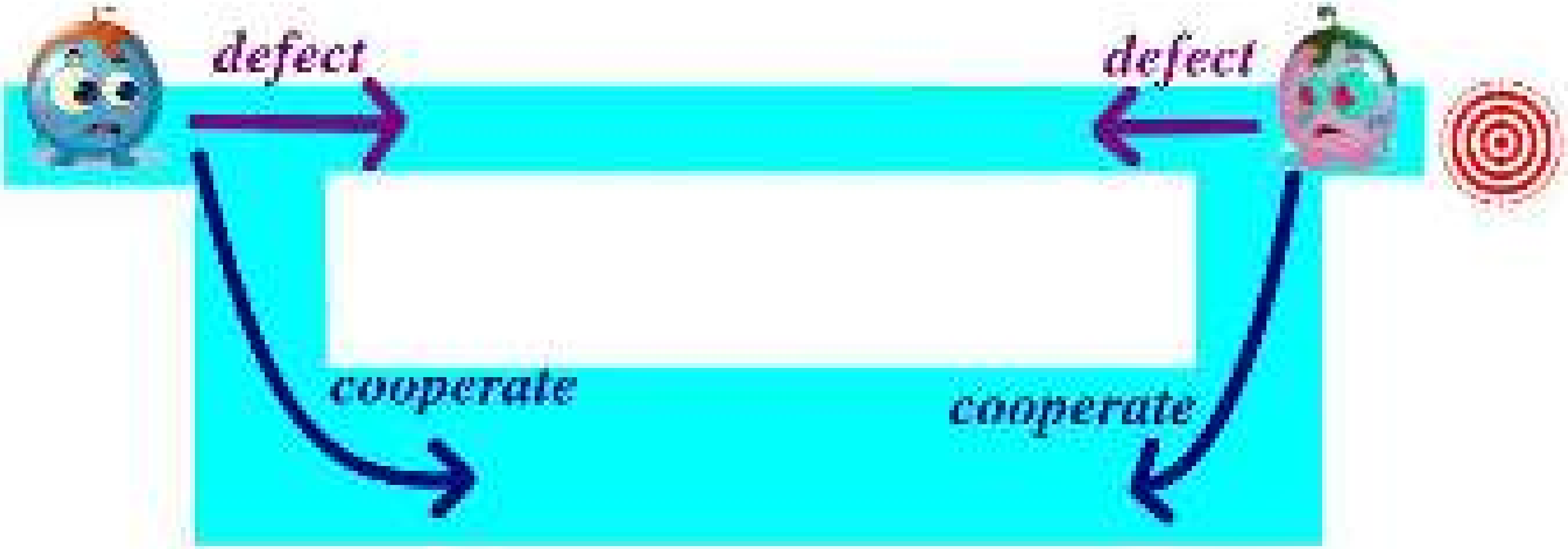
- Prisoners' dilemma (Tucker, 1950)

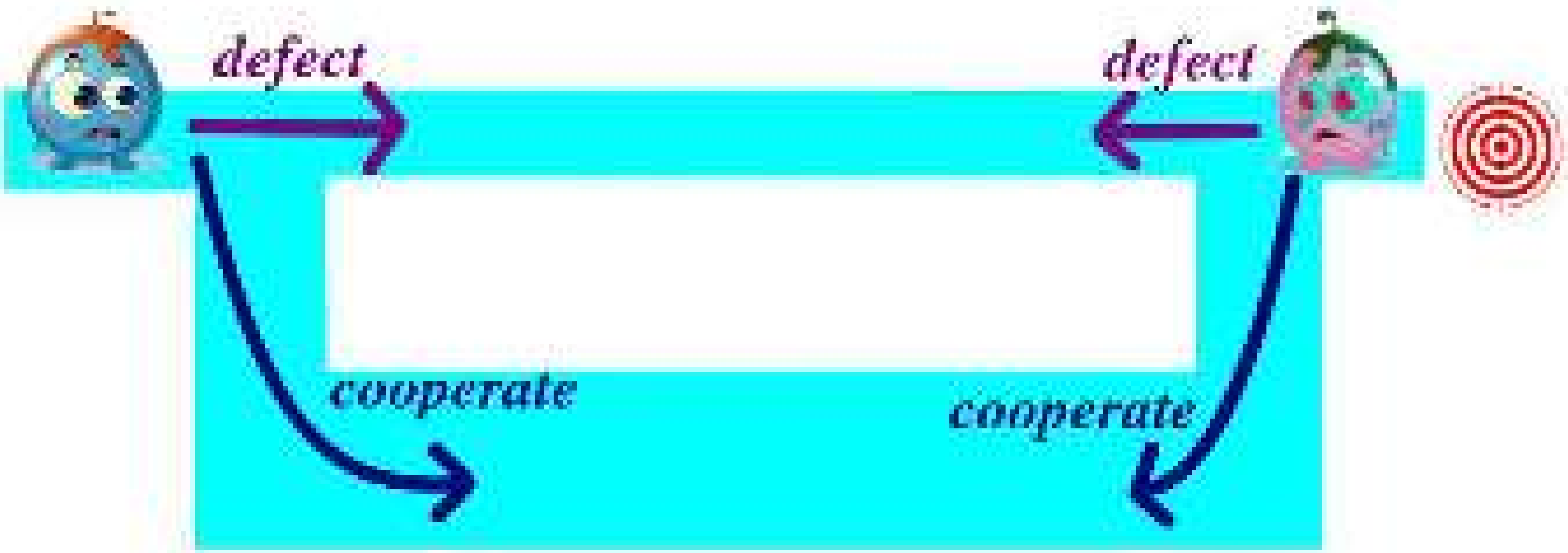


	Cooperate	Defect
Cooperate	1 1	0 10
Defect	10 0	5 5

Individual costs C (negative payoffs)

- s_j is a **best response** to s_k *iff* $s_j \in \arg\min_{s_j} C_j(s_j, s_k)$
- Here, although social optimum points to C, the prisoner's best response is always D.

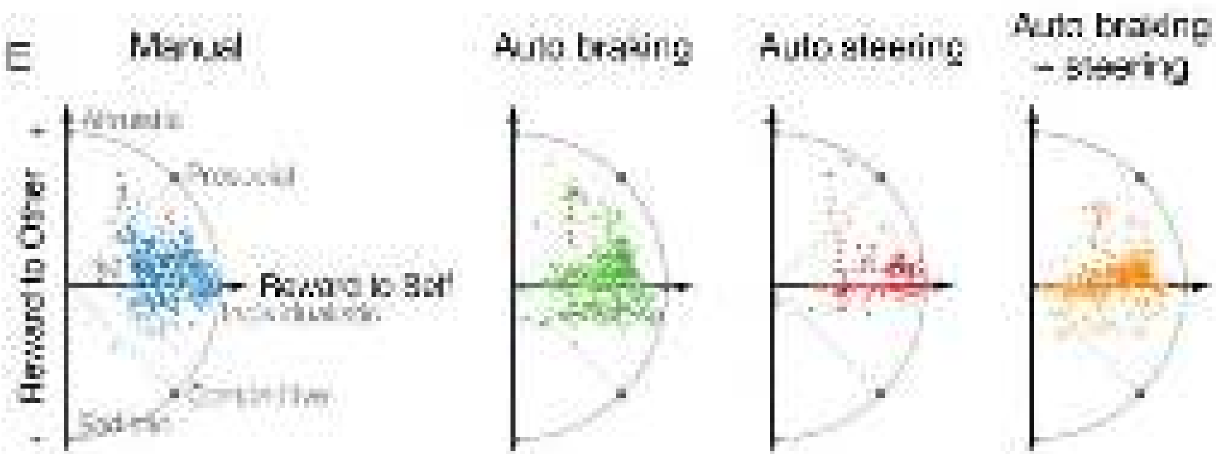

defect
defect
cooperate
cooperate



- Generalising to *N* players and to continuous space

$$C_j\left(\{\boldsymbol{r}_j\}_{t=0}^{T} \,\middle|\, \{\boldsymbol{r}_{-j}\}\right) = \int_0^T c\left(\boldsymbol{r}_j(\tau), \boldsymbol{v}_j(\tau), \{\boldsymbol{r}_{-j}(\tau)\}\right) d\tau + C_j^T\left(\boldsymbol{r}_j(T)\right)$$

where $\boldsymbol{r}_{-j} \doteq \{\boldsymbol{r}_1, \ldots, \cancel{\boldsymbol{r}_j}, \ldots, \boldsymbol{r}_N\}$



Shirado et al., *PNAS* (2023)

- $(s_1^*,\dots,s_N^*)$ forms a **Nash equilibrium**

  iff every s_k^* is a best response to s_{-k}^*

  i.e.

$$
\begin{array}{lll}
\forall s_1 \in S_1, & C_1(s_1, s_{-1}^*) \geq & C_1(s_1^*, s_{-1}^*) \\
\forall s_2 \in S_2, & C_2(s_2, s_{-2}^*) \geq & C_2(s_2^*, s_{-2}^*) \\
\vdots & \vdots &
\end{array}
$$

[Nash's theorem]

Every game with finite strategies has at least one Nash equilibrium (with either pure or mixed strategies)

- $(s_1^*,\ldots,s_N^*)$ forms a **Nash equilibrium**

  iff every s_k^* is a best response to s_{-k}^*

  i.e.

$$
\begin{aligned}
&\forall s_1 \in S_1, \quad C_1(s_1, s_{-1}^*) \geq C_1(s_1^*, s_{-1}^*) \\
&\forall s_2 \in S_2, \quad C_2(s_2, s_{-2}^*) \geq C_2(s_2^*, s_{-2}^*) \\
&\qquad\vdots \qquad\qquad\quad \vdots
\end{aligned}
$$

[Nash's theorem]

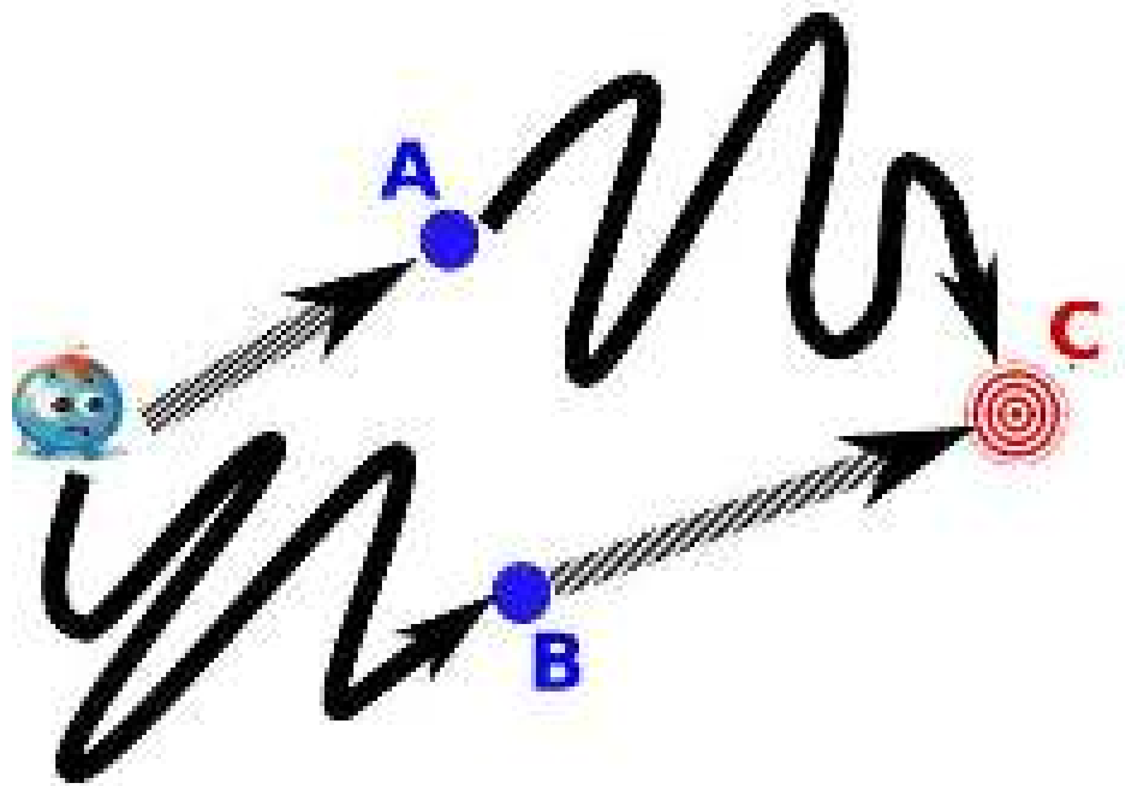
> Every game with finite strategies has at least one Nash equilibrium (with either pure or mixed strategies)

- Assuming a Common Knowledge of Rationality, namely

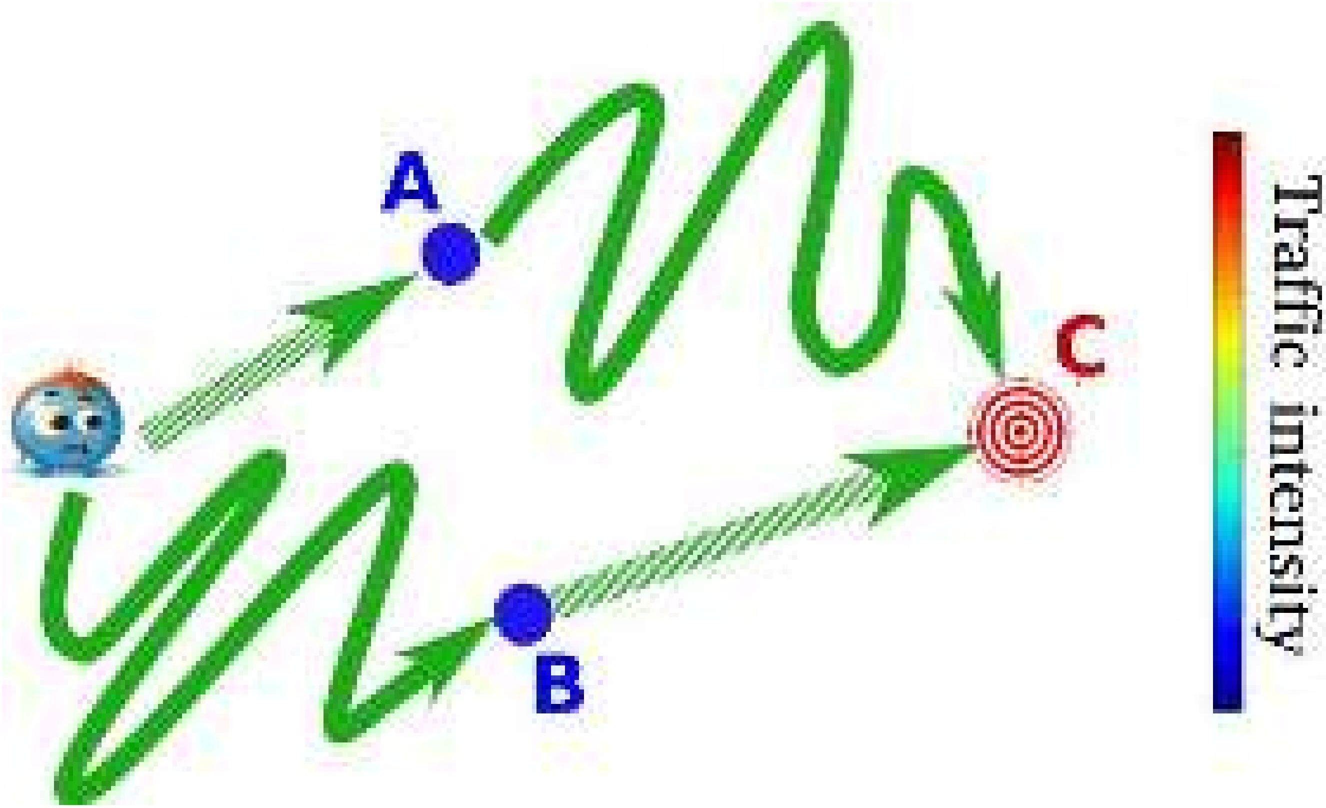
  I know that you will act rationally
  You know that I know that you will act rationally
  I know that you know that I know that you will act rationally
  ...

  the players will favour moves that are Nash equilibria.

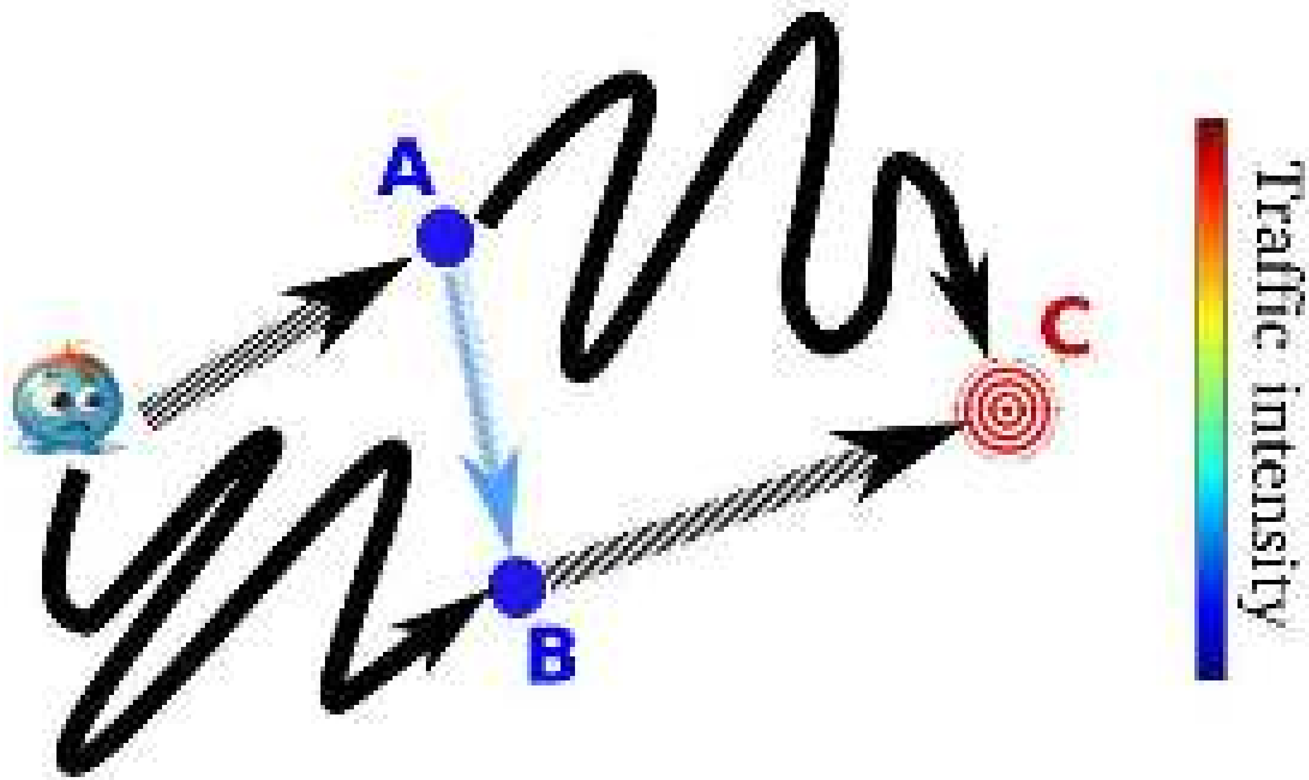
Game theory : *Braess's paradox*



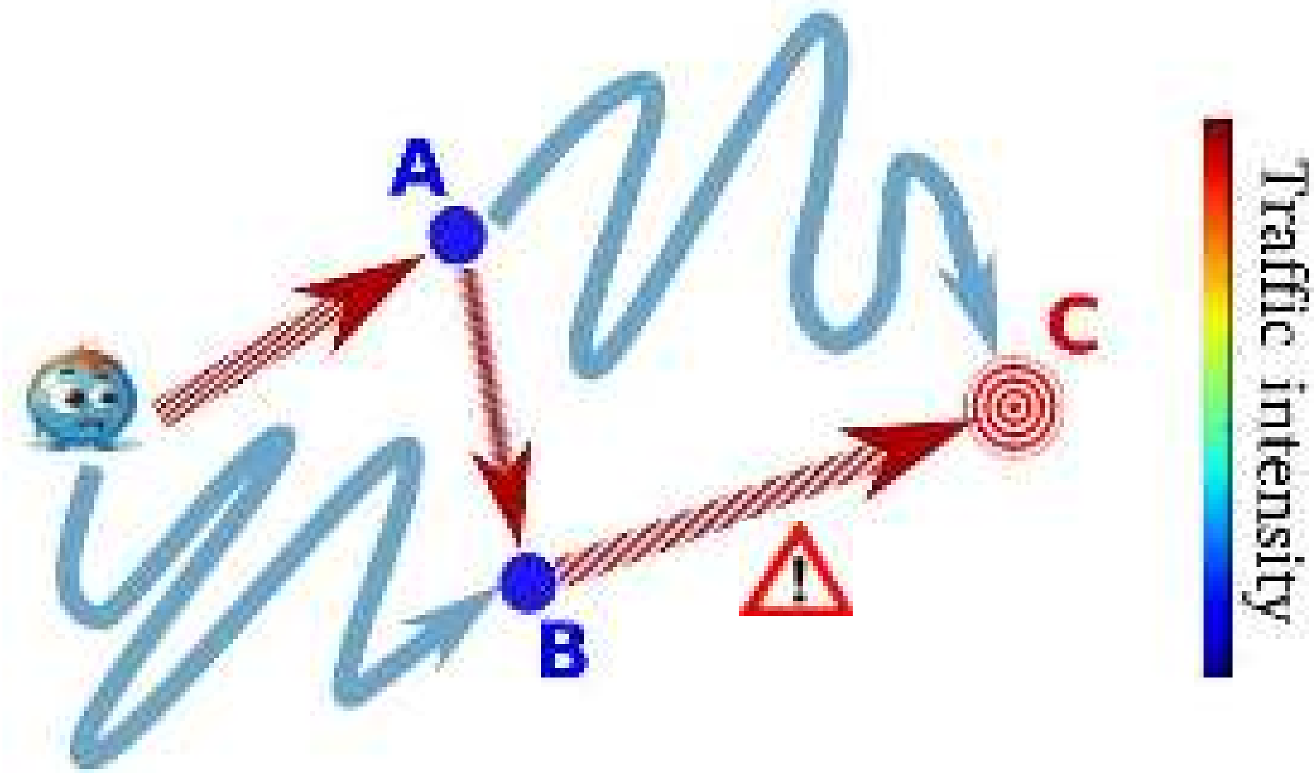
Game theory : *Braess's paradox*



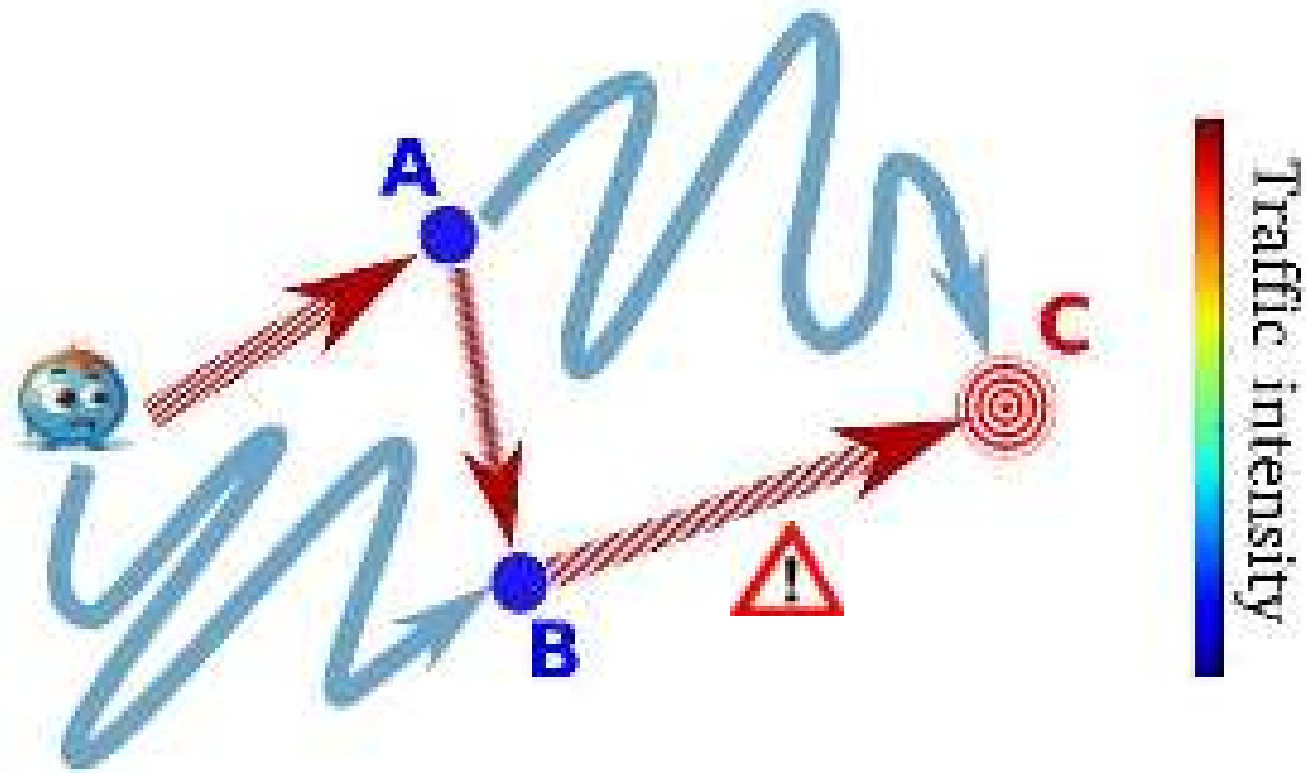
Game theory : *Braess's paradox*



Game theory : *Braess's paradox*



Game theory : *Braess's paradox*

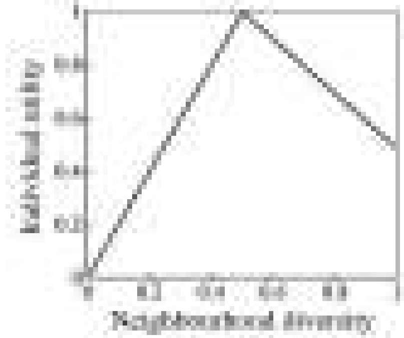


System optimum ≠ user equilibrium
(« *Wardrop's principles of equilibrium* »)

PNAS

Competition between collective and individual dynamics

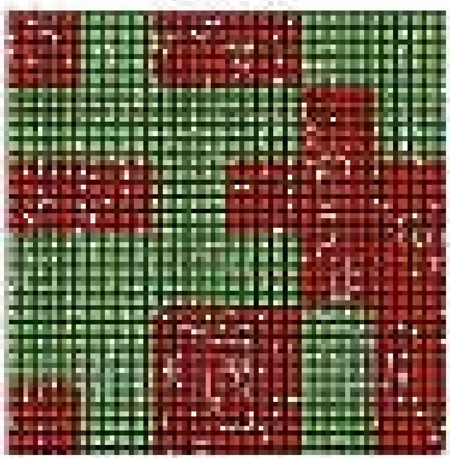
Sébastian Grauwin, Eric Bertin, Rémi Lemoy, and Pablo Jensen



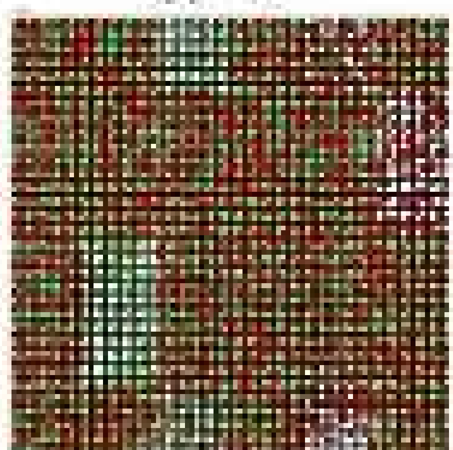
essential differences across disciplines: Although physical particles are assumed to optimize the global energy, economic agents maximize their own utility. Here, we solve exactly a Schelling-

$$\mathcal{G} = \underbrace{\Delta w}_{\text{individual payoff}} + \alpha\,(\underbrace{\Delta W}_{\text{collective benefit}} - \Delta w)$$

$\alpha = 0$



$\alpha = 1$



Grauwin et al., *PNAS* (2009)

iLM

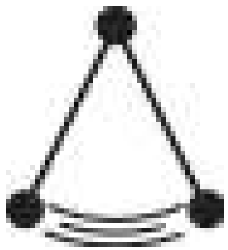
Lyon 1

CNRS

- Wildly different steady states...
  ... but what about the dynamics ?

  E.g. pendulum with infinitesimal friction

$$\ddot{x} = -\epsilon\dot{x} - \omega^2 x$$



- Wildly different steady states…
  … but what about the dynamics ?

- Restore some dynamics here

$$\forall j \in [1, N], \; \eta \dot{s}_j = -\left.\frac{\partial C_j}{\partial s_j}\right|_{s_{-j}}$$

- Wildly different steady states...
  ... but what about the dynamics ?

- Restore some dynamics here

$$\forall j \in [1, N],\ \eta \dot{s}_j = -\left.\frac{\partial C_j}{\partial s_j}\right|_{s_{-j}}$$

... similar to overdamped dynamics of a physical system with conservative interactions $U(s_1,\ldots,s_N)$:

$$\forall j \in [1, N],\ \eta \dot{s}_j = -\left.\frac{\partial U}{\partial s_j}\right|_{s_{-j}}$$

except that every agent has their own ‘potential’.
Thus, interactions are not necessarily *reciprocal* :

$$\frac{\partial}{\partial s_k}\frac{\partial U}{\partial s_j} = \frac{\partial}{\partial s_j}\frac{\partial U}{\partial s_k} \quad \text{but} \quad \frac{\partial}{\partial s_k}\frac{\partial C_j}{\partial s_j} \overset{?}{\neq} \frac{\partial}{\partial s_j}\frac{\partial C_k}{\partial s_k}$$

Potential games

- There is a category of games for which each choice can be ascribed to a global potential

$$\forall j \in [1, N],\ C_j(s'_j, s_{-j}) < C_j(s''_j, s_{-j}) \Leftrightarrow U(s'_j, s_{-j}) < U(s''_j, s_{-j})$$

The class of potential games coincides with the class of congestion games

Monderer & Shapley, *Games and Economic Behavior* (1994)

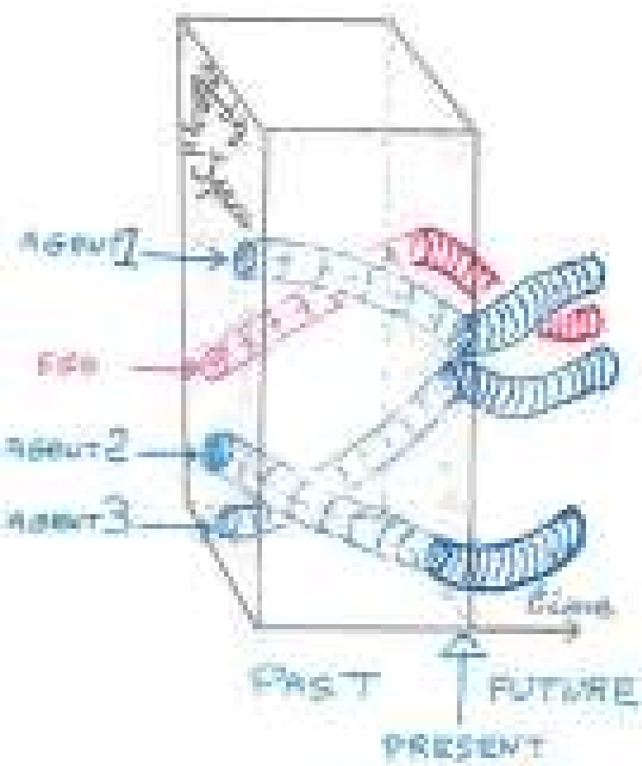
Rosenthal, *Int'l J. of Game Theory* (1973)

The equivalent potential of a potential game need not be the collective cost (or social welfare) !

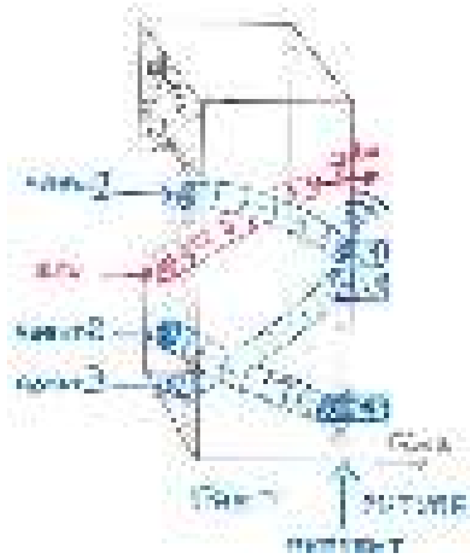

Back to crowd dynamics

$$C_i\left(\{r_i\}_{t=0}^{T}\,\middle|\,\{r_{-i}\}\right) = \int_0^T c\left(r_i(\tau), u_i(\tau), \{r_{-i}(\tau)\}\right) d\tau + C_i^T\left(r_i(T)\right)$$

$$c_i^{(\tau)} = c_{\text{control}}\left(u_i(\tau)\right) + \sum_{k \in PV_i} V_{\text{tip}}\left(r_{ik}(\tau)\right) + \dots$$

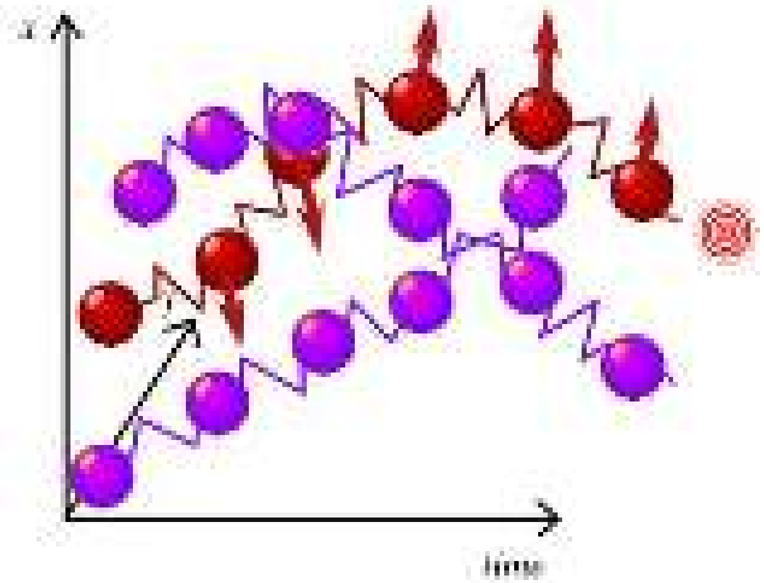


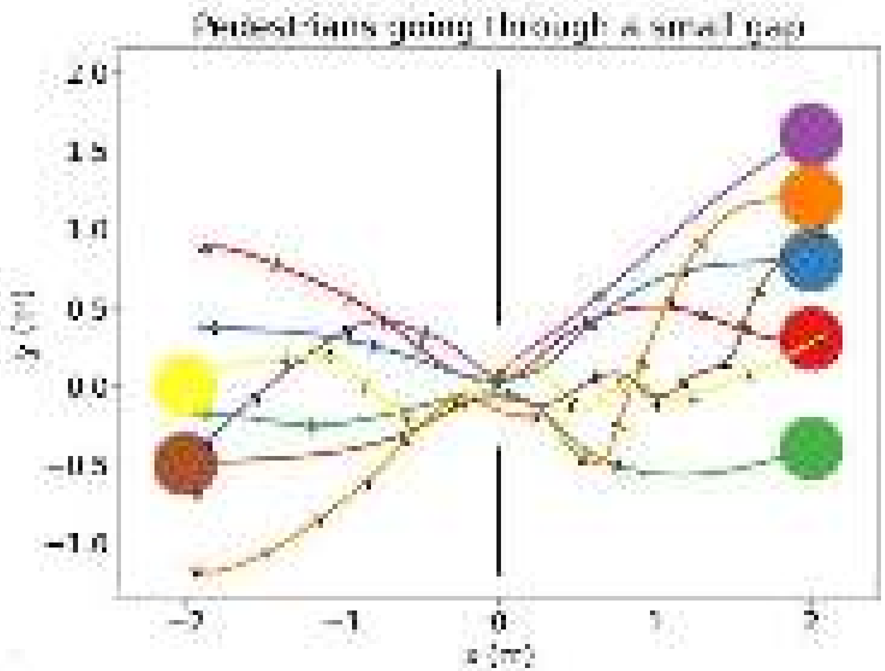
Game theory



Reactive model

Space-time trajectories look like stretched polymers



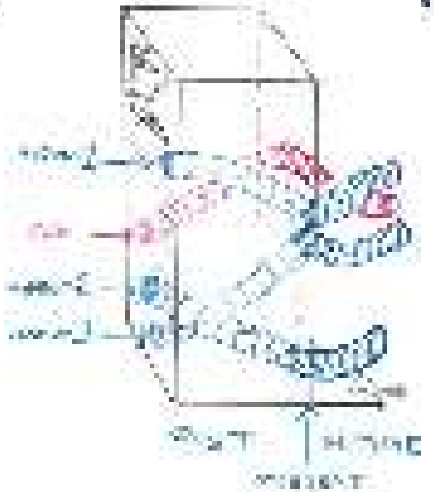


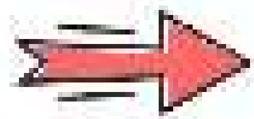
~ Concept of « Time Elastic Band » in robotics

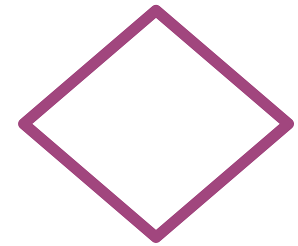
Chung, Youssef & Roidl, *IEEE Conf. On Robotics and Automation* (2022)

With Alexis RAULIN FOISSAC

Mean-field games

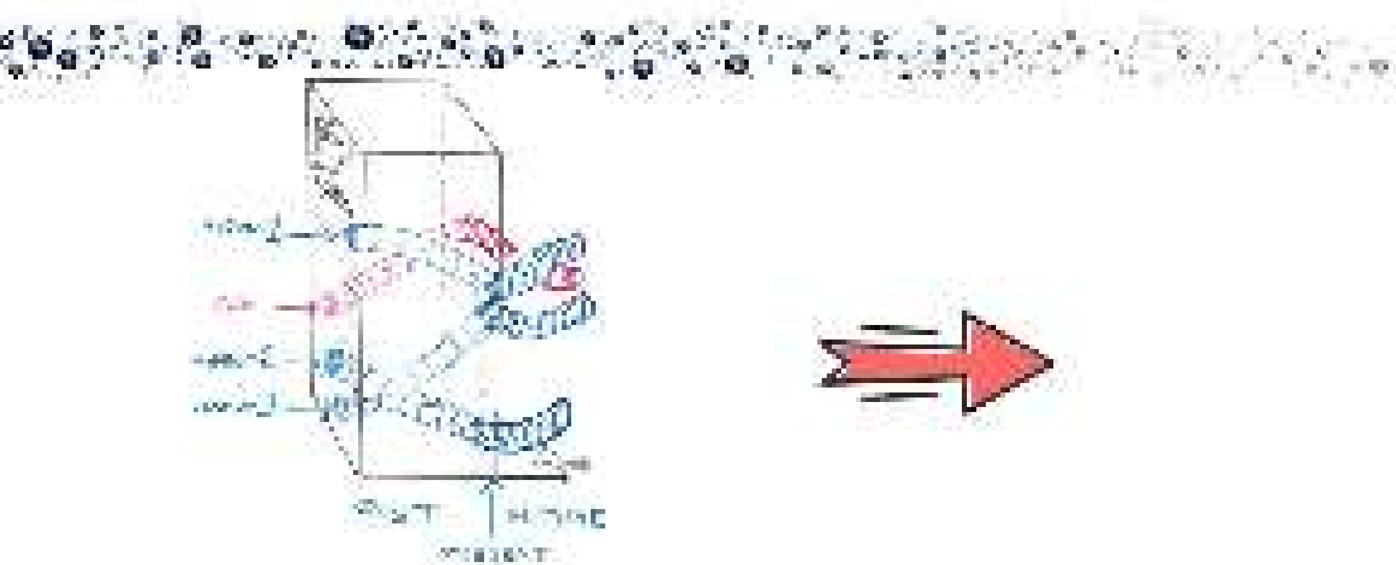






Bonnemain, ..., Ullmo, *Phys. Rev. E* (2023)

Mean-field games



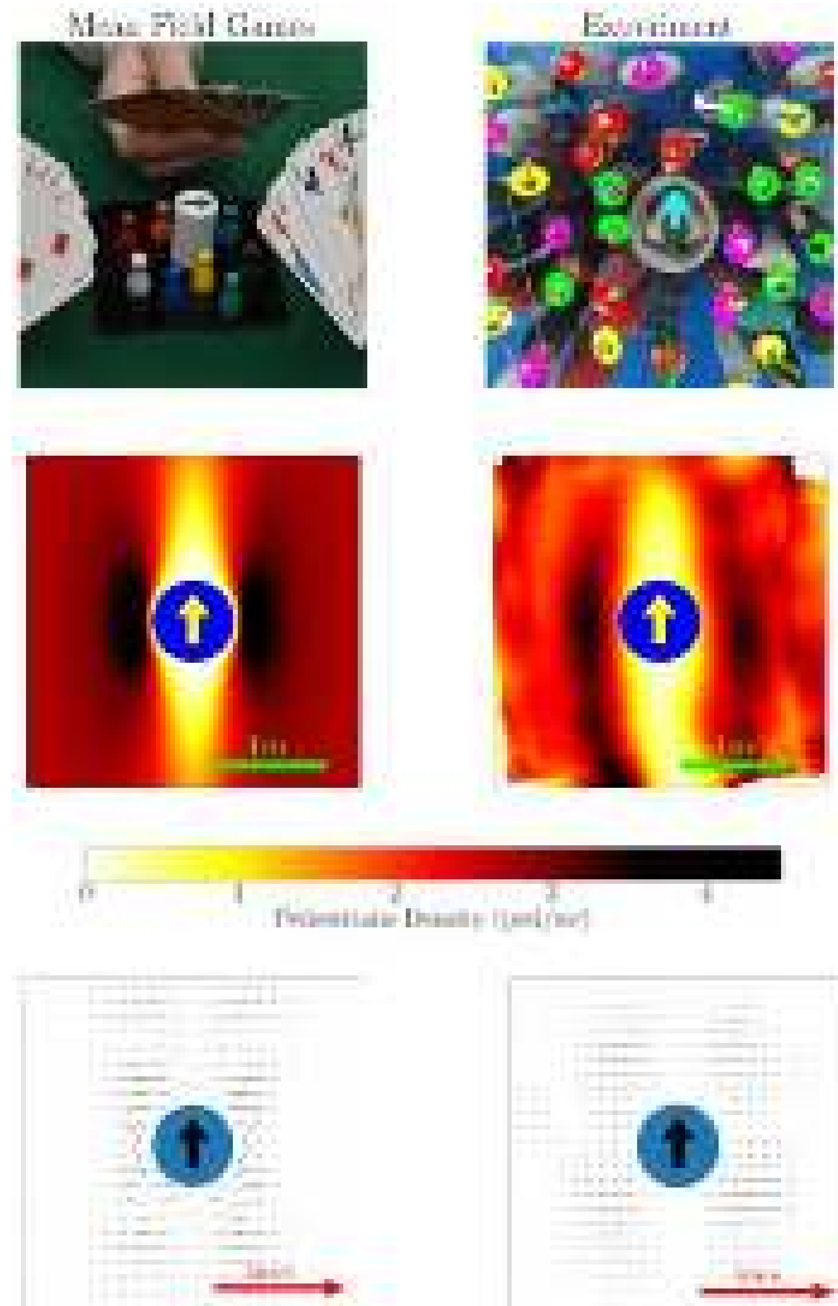
	Individual	Mean field
Variable	Positions $\{r_j\}$	Density $\rho(t,r) = \frac{1}{N}\left\langle \sum_j \delta(r - r_j(t)) \right\rangle_{\text{noise}}$
Dynamics	Langevin $dr_j = v_j\,dt + \sigma dW_j$	(Fokker-Planck | *forwards*) $\frac{\partial \rho}{\partial t} = -\frac{1}{\mu}\nabla\cdot[\rho\nabla u] + \sigma^2\Delta\rho$
Choices	Running cost $c_j^{\text{run}} = \frac{\mu}{2}v_j^2 - g\sum_k \delta(r_j - r_k) + U(r_j)$	(Hamilton-Jacobi-Bellman | *backwards*) $\frac{\partial u}{\partial t} = \frac{1}{2\mu}[\nabla u]^2 - \frac{\sigma^2}{2}\Delta u - g\rho - U$

Bonnemain, ..., Ullmo, *Phys. Rev. E* (2023)

Type of model

Density field around intruder

Velocity field around intruder

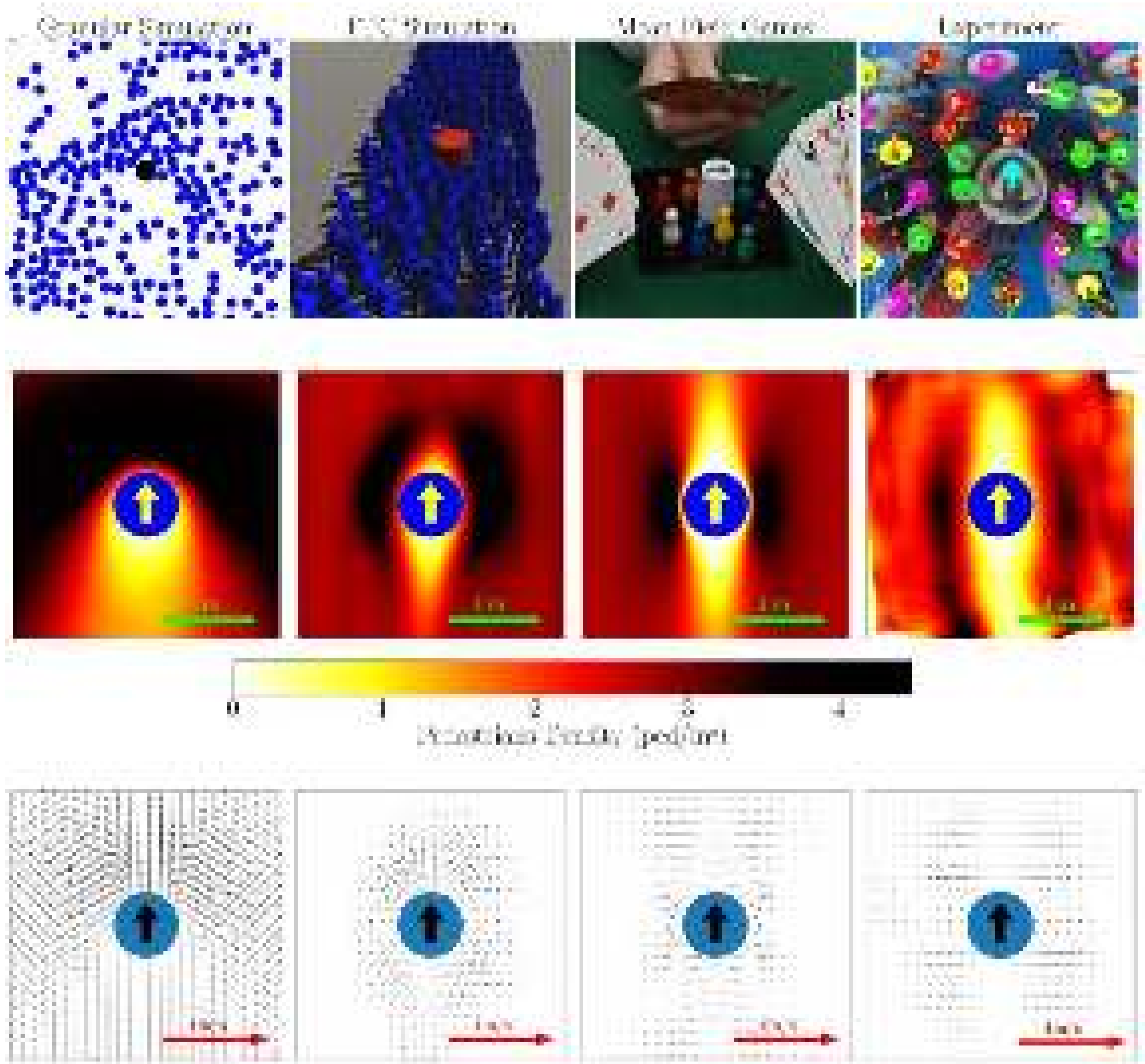


How do other types of models perform ?

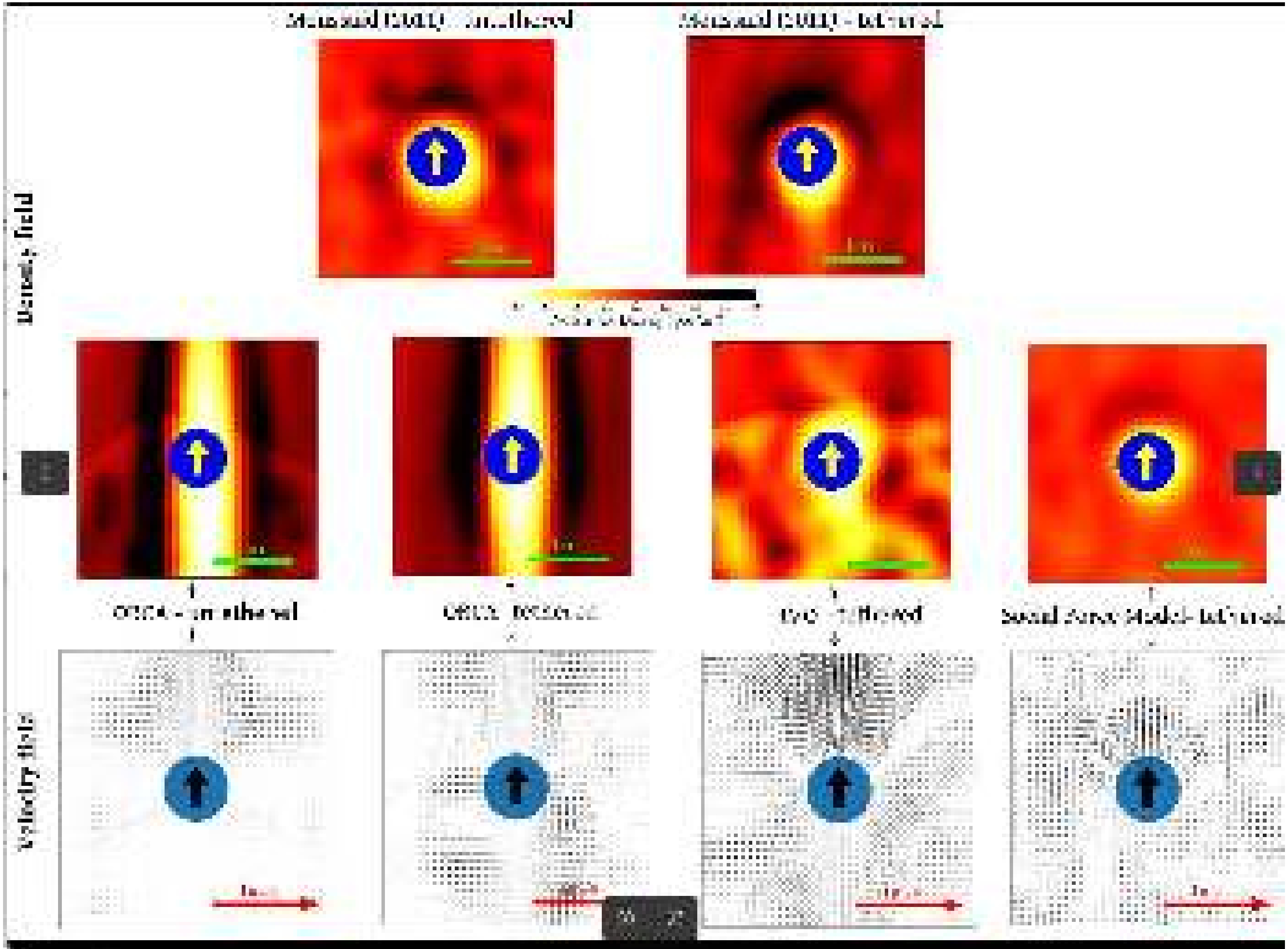
Type of model

Density field around intruder

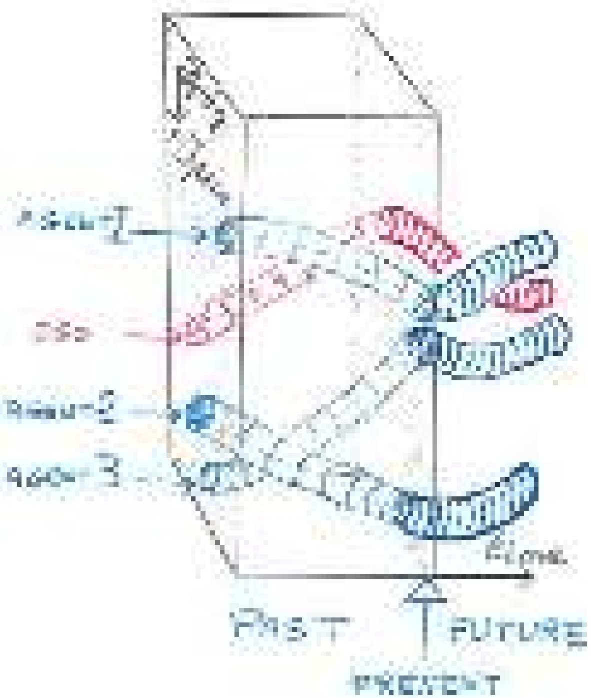
Velocity field around intruder

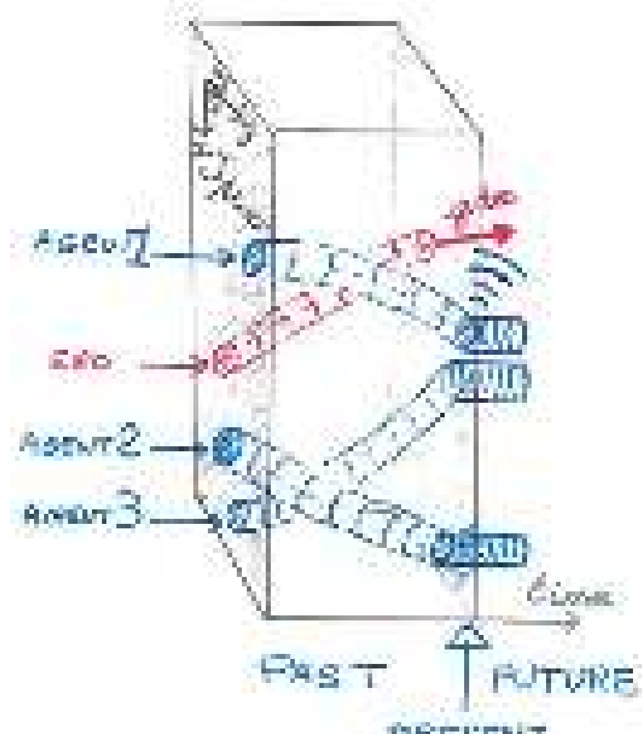


How do (yet) other types of models perform ?

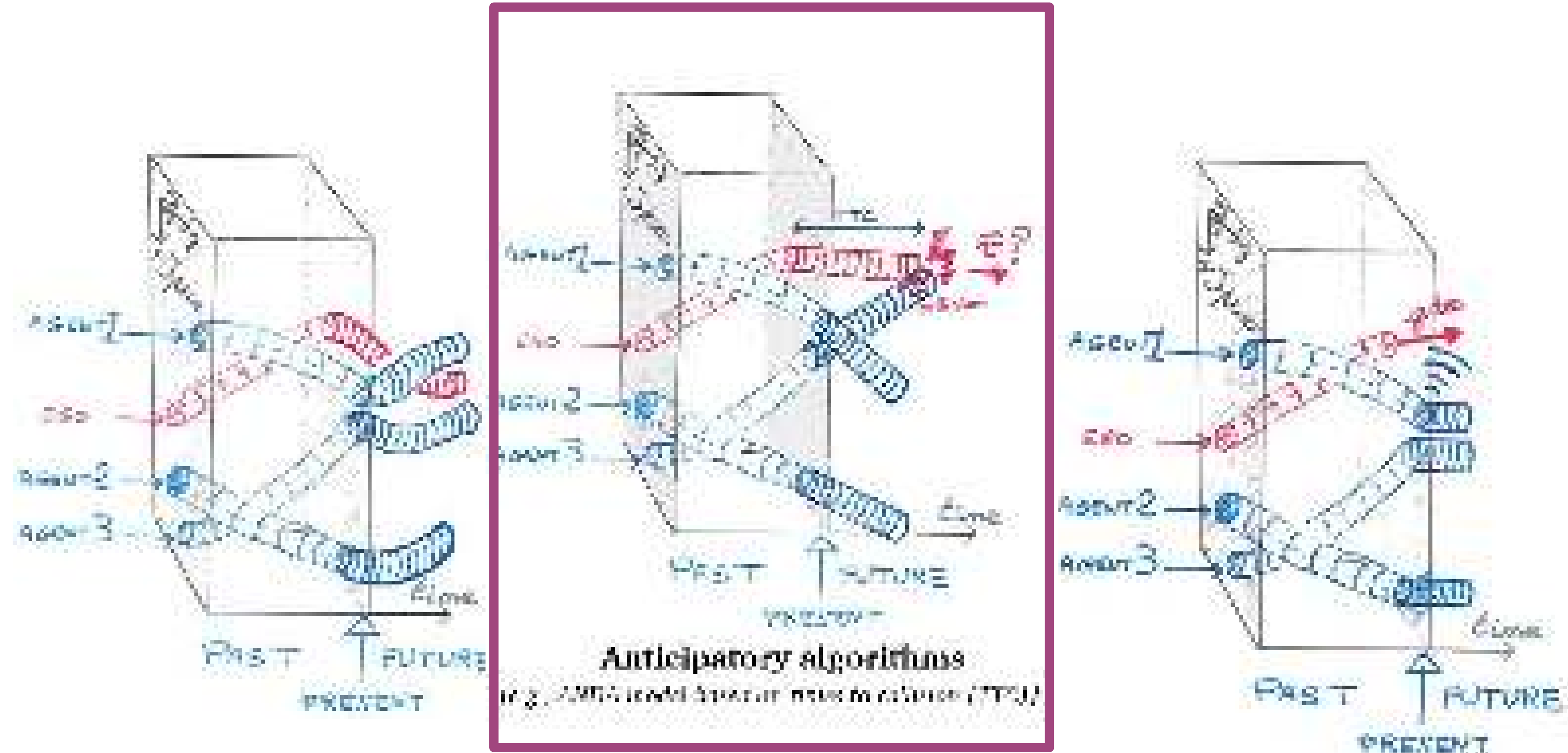


Anticipation of collisions





Anticipation of collisions



Anticipatory algorithms

Collision avoidance in the social psychological literature since the 1970s

Wolff (1973)
Collett and Marsh (1974)

« Velocity obstacle » approach : *bar velocities that lead to a future collision*

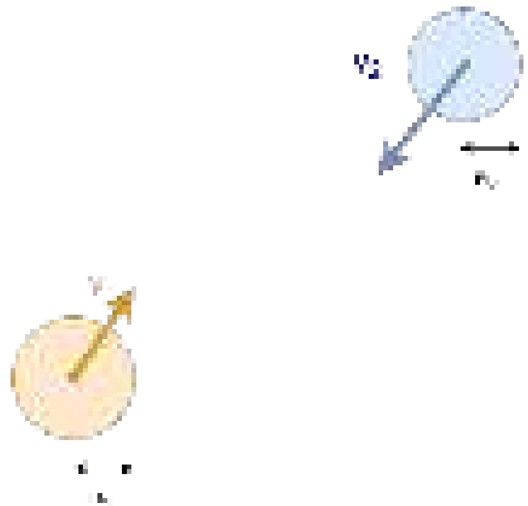
J. van Den Berg, Ming Lin, S. Guy (USA)

S. Faure & B. Maury (Orsay)

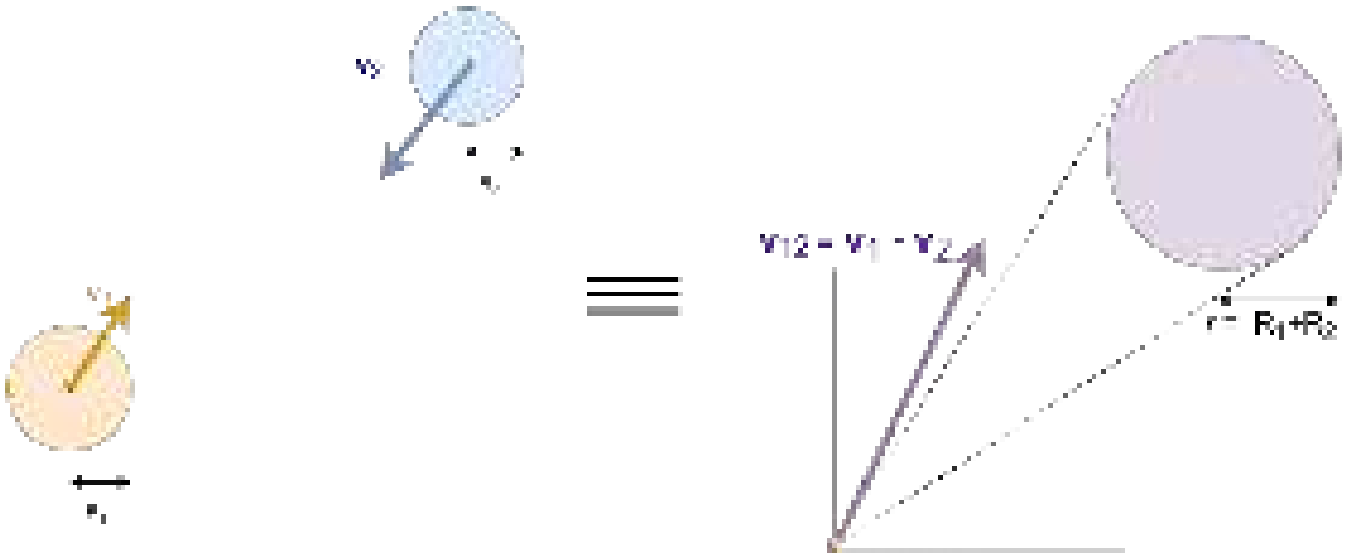
A.-H. Olivier, J. Pettré (Rennes)

and many others...

Anticipated time to collision (TTC)

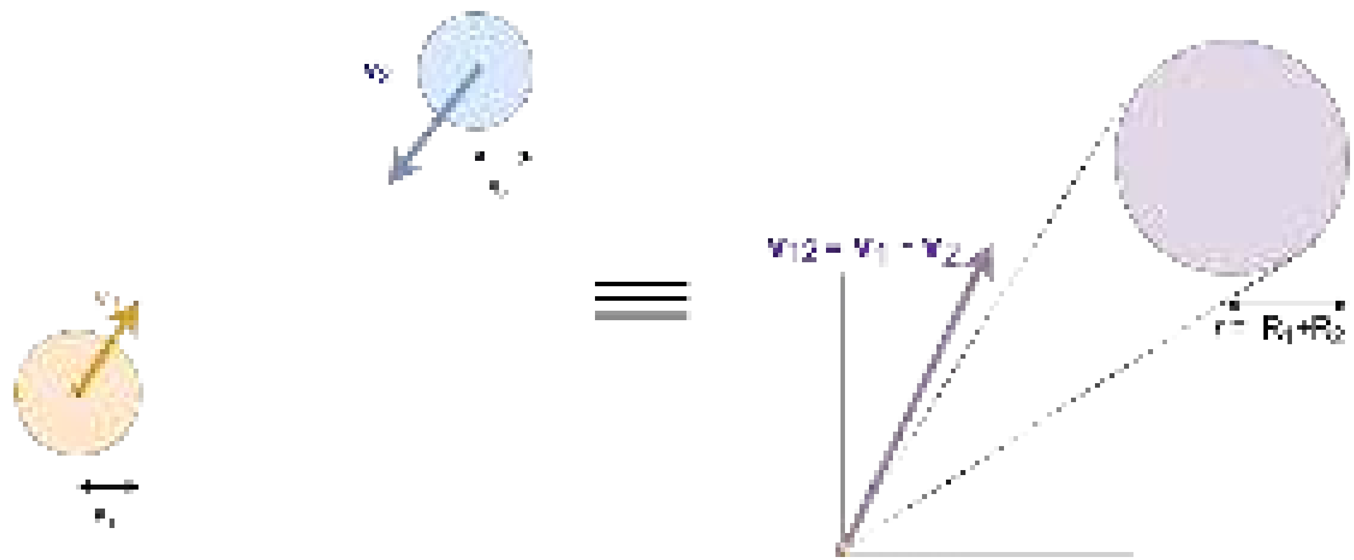


Anticipated time to collision (TTC)



Karamouzas, Skinner, Guy, *Phys. Rev. Lett.* (2014)

Anticipated time to collision (TTC)



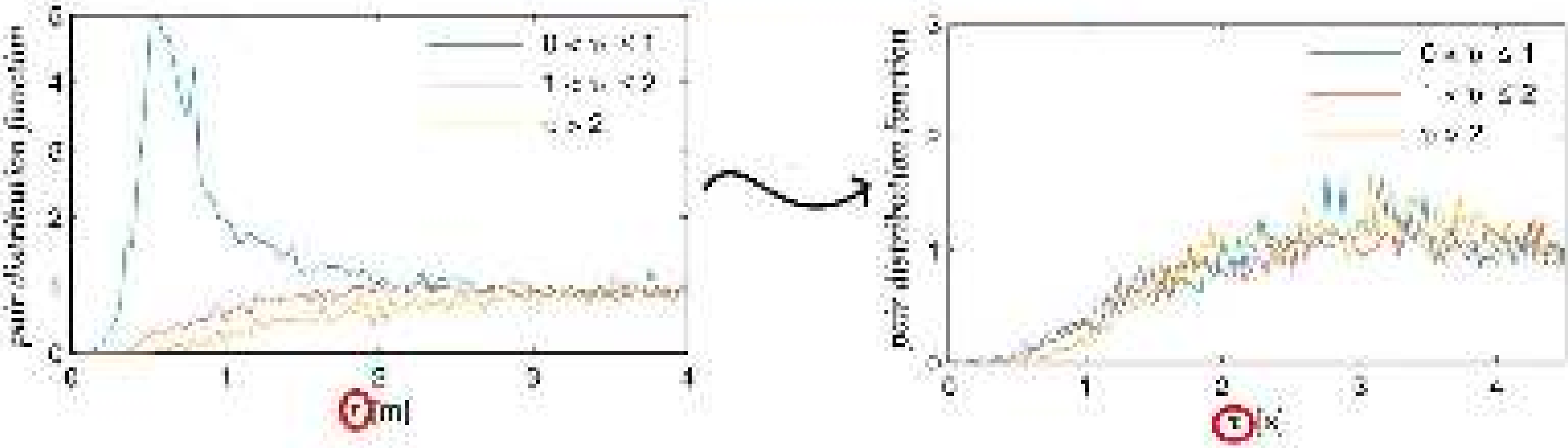
Time to collision

$$\tau = \frac{-\mathbf{x}_{ij}\cdot\mathbf{v}_{ij} - \sqrt{(\mathbf{x}_{ij}\cdot\mathbf{v}_{ij})^2 - \|\mathbf{v}_{ij}\|^2(\|\mathbf{x}_{ij}\|^2 - r^2)}}{\|\mathbf{v}_{ij}\|^2}$$

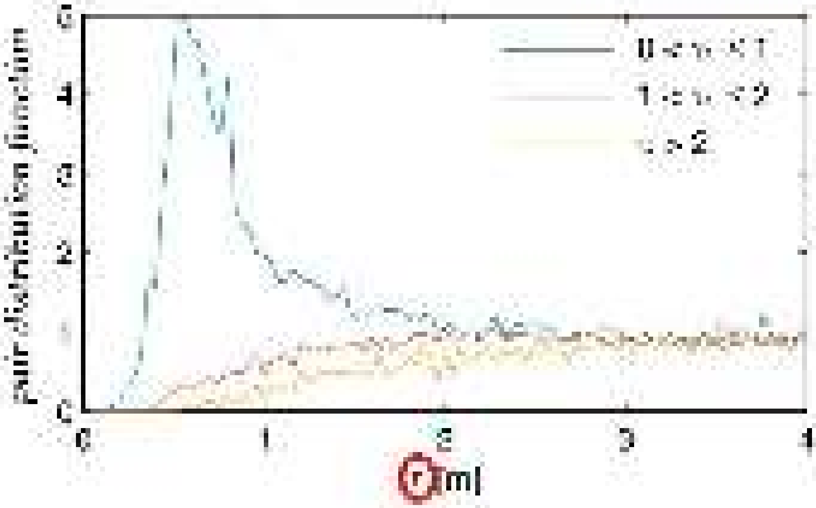
Karamouzas, Skinner, Guy, *Phys. Rev. Lett.* (2014)

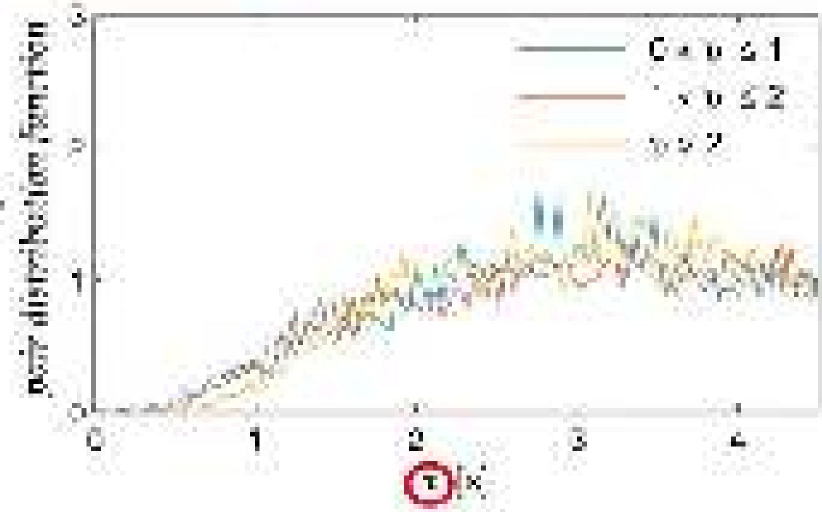
Empirical study of the times to collision

Snapshots from the pedestrian database



Karamouzas, Skinner, Guy, *Phys. Rev. Lett.* (2014)



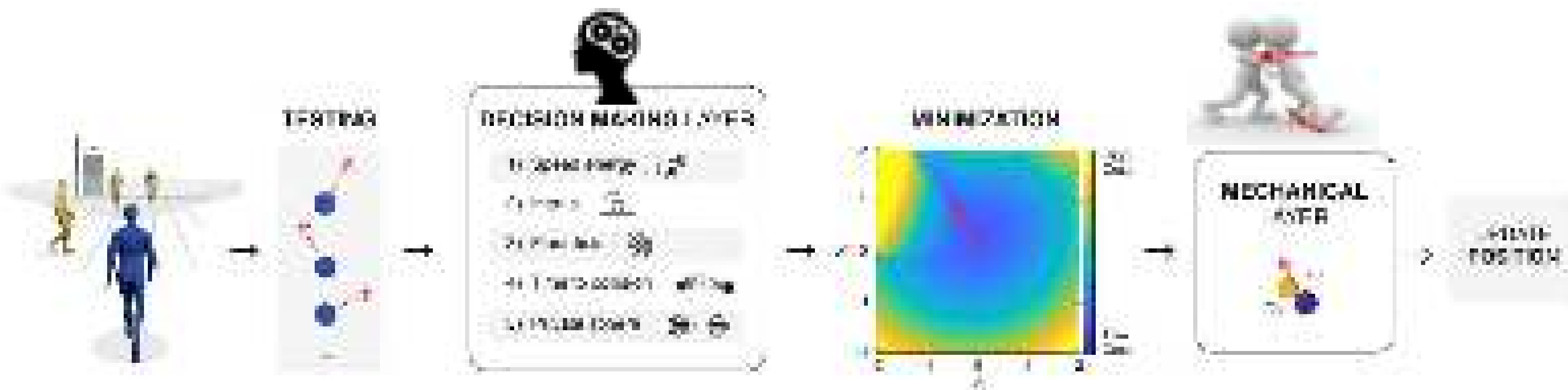


Using the formula from liquid-state theory,

$$g(\tau) \propto \exp\left(\frac{-E(\tau)}{E_0}\right) \rightsquigarrow E(\tau) \propto \tau^{-2} \exp\left(\frac{-\tau}{\tau_0}\right)$$

Karamouzas, Skinner, Guy, *Phys. Rev. Lett.* (2014)

The Anticipatory Dynamics model (ANDA)



$$v^* = \underset{v \in \mathbb{R}^2}{\operatorname{argmin}} E[v]$$

Echeverría-Huarte & Nicolas , *Transp. Res. C* (2023)

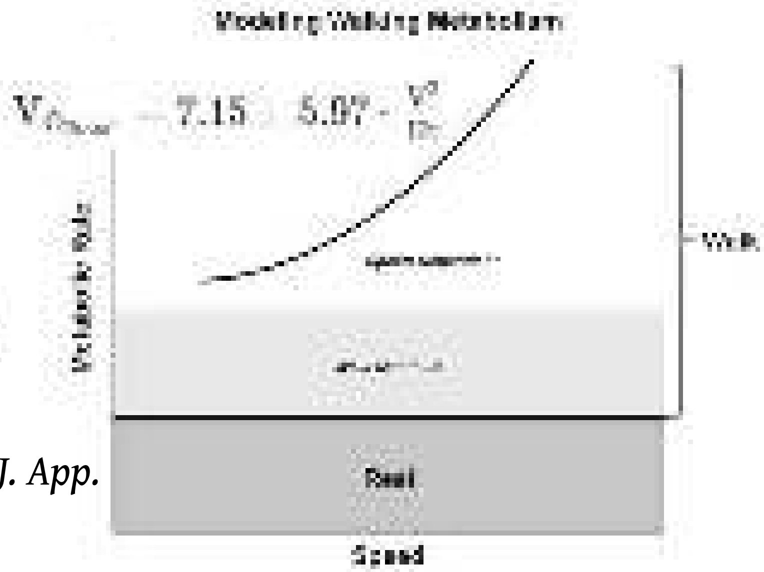
$$C_i\left[\{r_i\}_{t=0}^{T}\right] \sim E(v) = e_{\text{biomech}}(v) + \cdots + \text{TTC penalty} + \underbrace{u\left(r_i + v\delta t\right)}_{\text{Floor field} \sim \text{value function}}$$

Also see Van Toll, ..., Pettré, *Symposium on Interactive 3D...* (2020)

$$C_j\left[\{r_j\}_{t=0}^{T}\right] \rightsquigarrow E(v) = e_{\text{biomech}}(v) + \cdots + \text{TTC penalty} + \underbrace{u\left(r_j + v\delta t\right)}_{\text{Floor field} \sim \text{value function}}$$

$$E(r,v) = E^{target} + \boxed{E^{speed}} + E^{TTC} + E^{Repulsion} + E^{penal}$$

Biomechanical cost of walking



Ludlow & Weyand, *J. App. Physiology* (2016)

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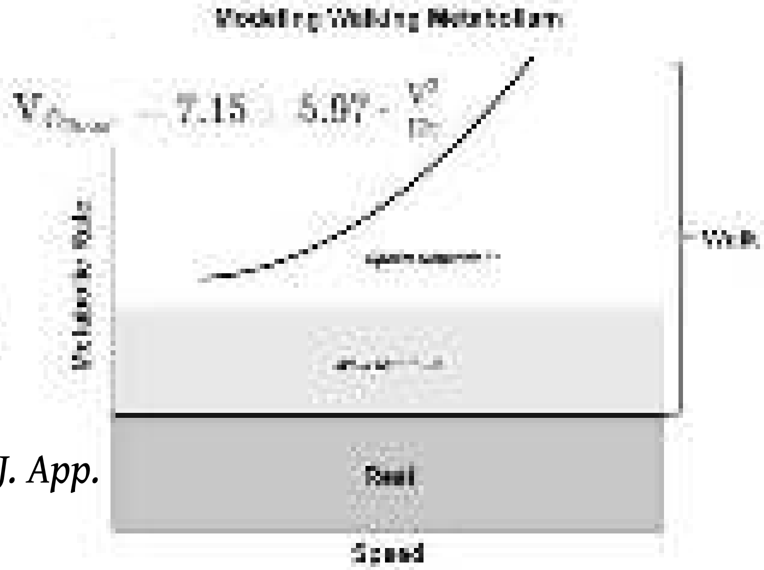
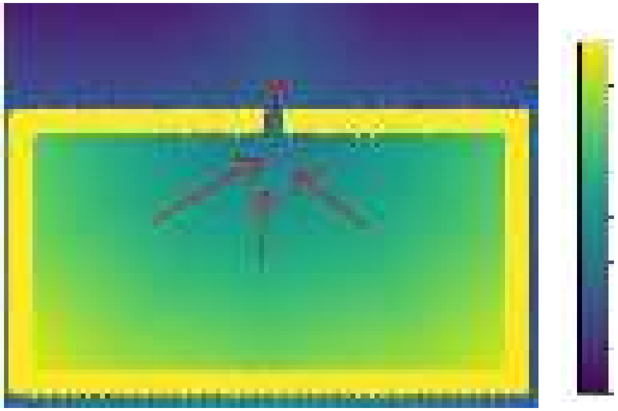


Ludlow & Weyand, *J. App. Physiology* (2016)

Driving term towards target



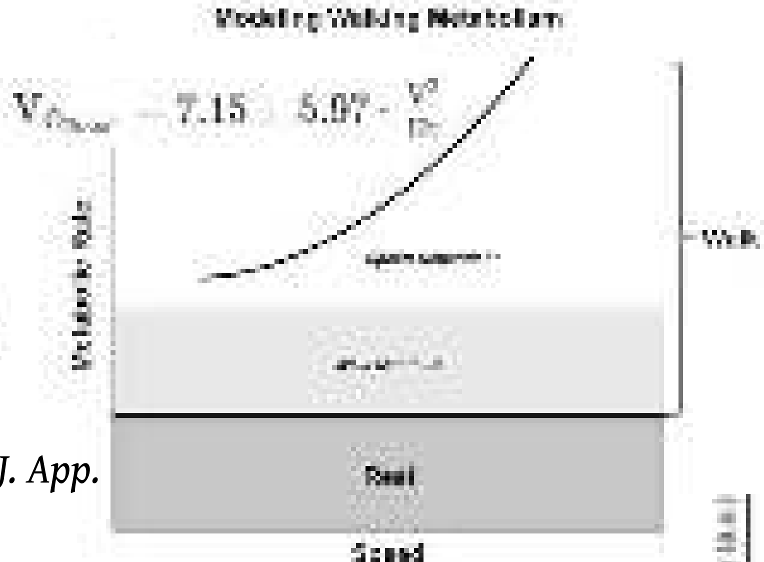
Floor field

(~ value function)

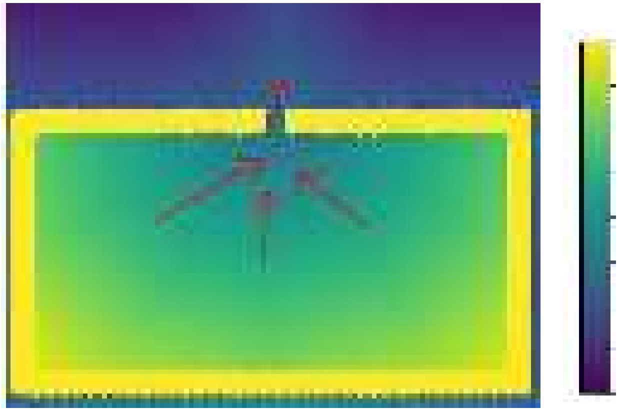
$$E(r,v) = \boxed{E^{target}} + \boxed{E^{speed}} + E^{TTC} + E^{Repulsion} + E^{penal}$$

Biomechanical cost of walking

Driving term towards target

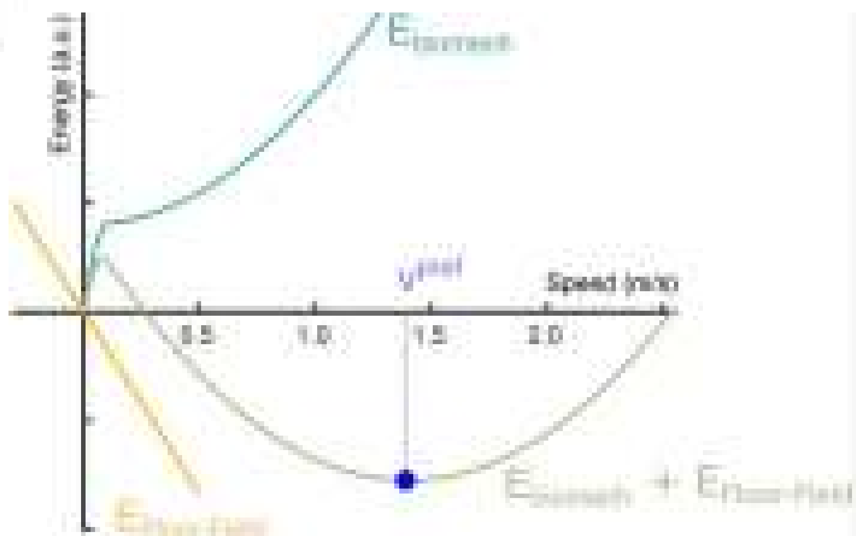


Ludlow & Weyand, *J. App. Physiology* (2016)



Floor field

$\Rightarrow$



$$E(r,v) = E^{target} + E^{speed} + E^{TTC} + E^{Repulsion} + \boxed{E^{penal}}$$

Penalty for abrupt changes in velocity

$$E^{penal} \propto \left| v' - v(t) \right|^2$$

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Desire to preserve one's private space

E.T. Hall, *The hidden dimension* (1963)

Here, repulsion grows as
1/distance in the private zone

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E.T. Hall, *The hidden dimension* (1963)

Here, repulsion grows as
1/distance in the private zone

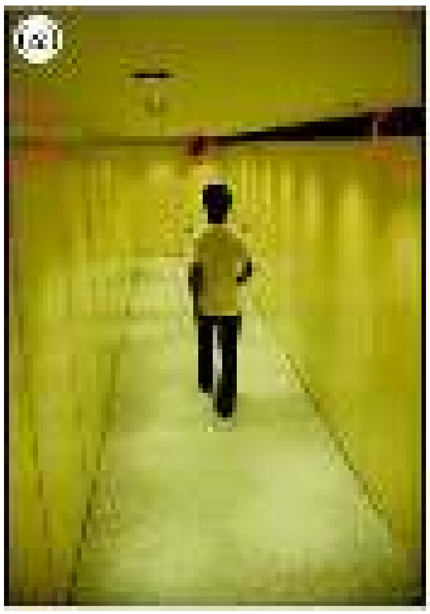
Anticipate and avoid future collisions

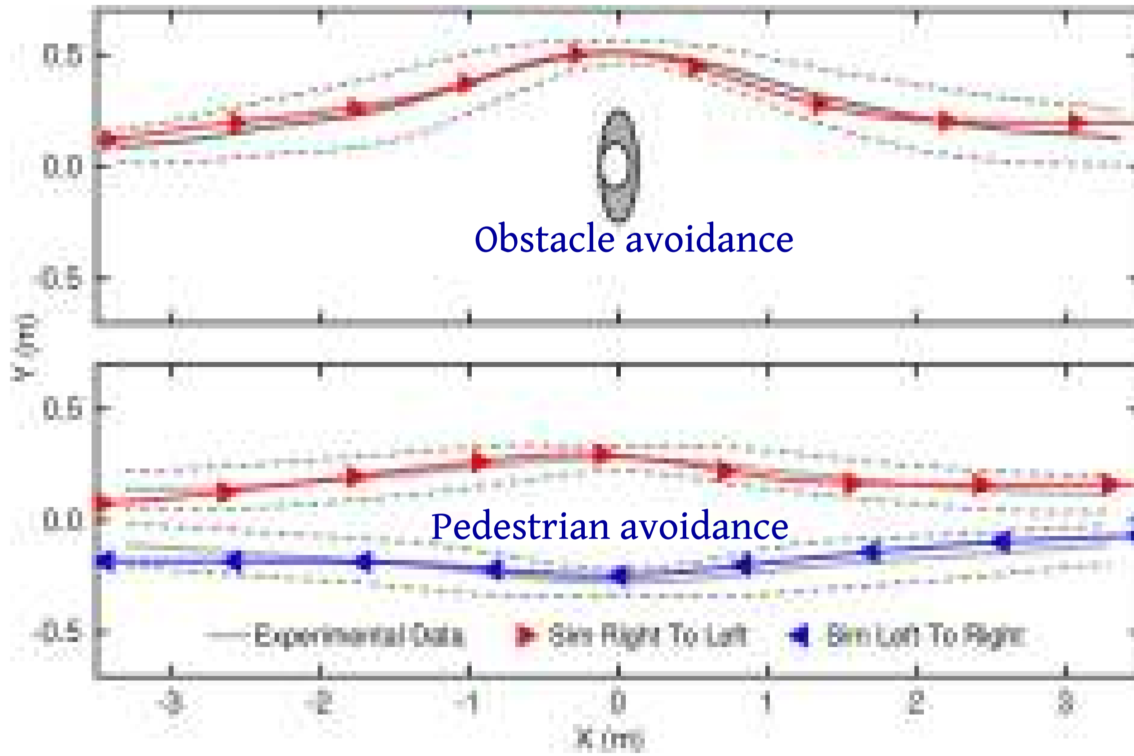
Time to collision

Lyon 1

Avoidance maneuvers

Thus calibrated, the model immediately reproduces one-on-one avoidance manoeuvres in a quantitative way





Experimental data from :
Moussaid et al., *Proc. Roy. Soc. B* (2009)

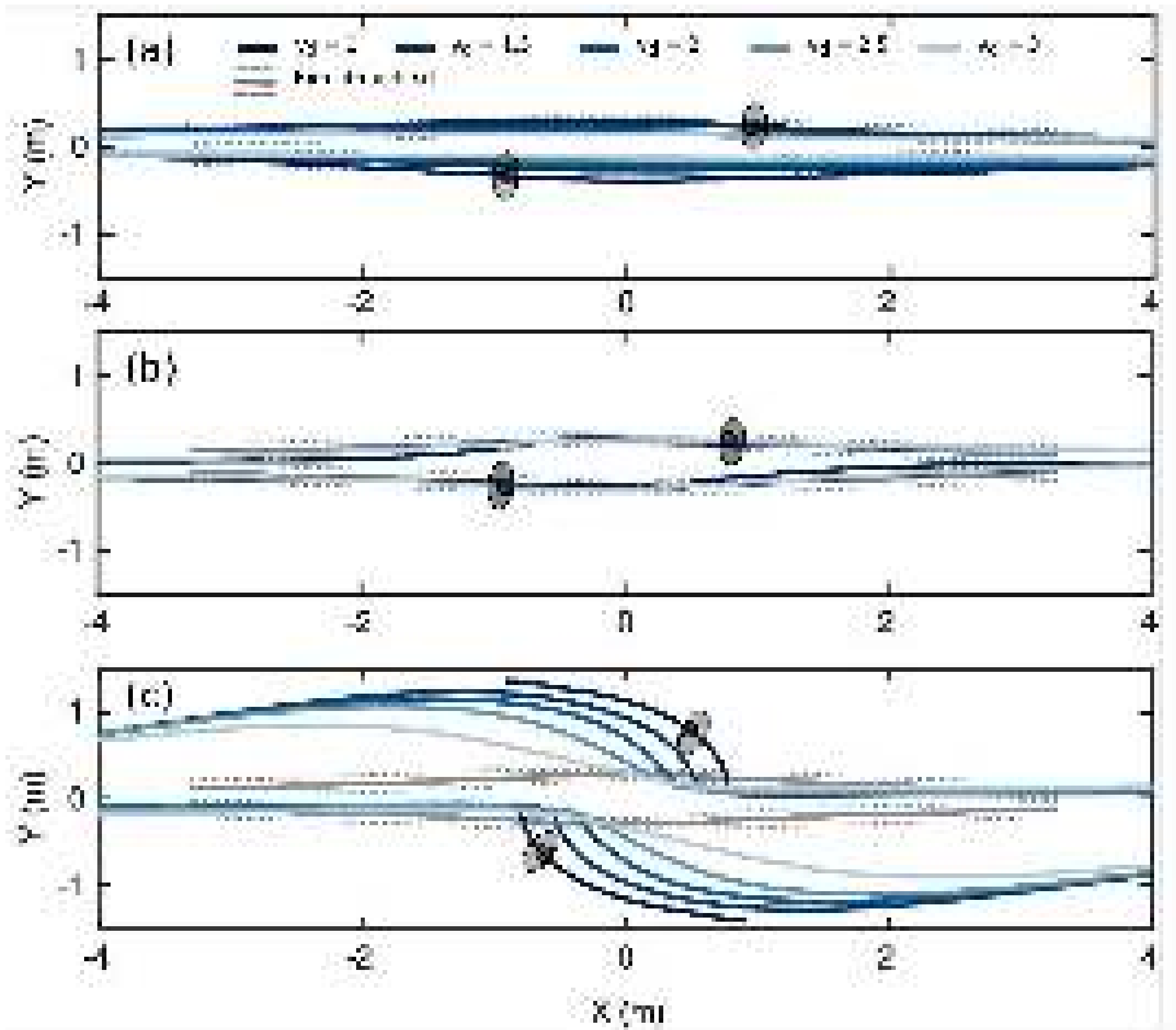
With Iñaki Echeverría_Huarte

Avoidance manoeuvre

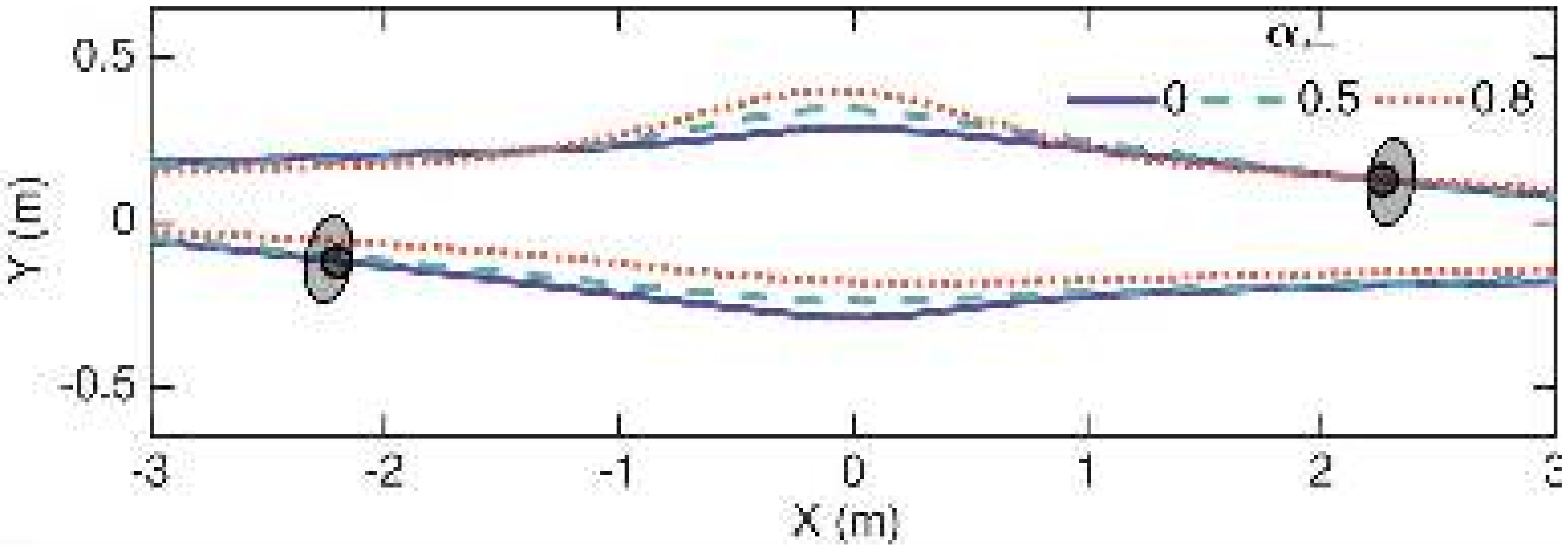
Game theory

TTC-based interactions

Distance-based interactions (SFM)



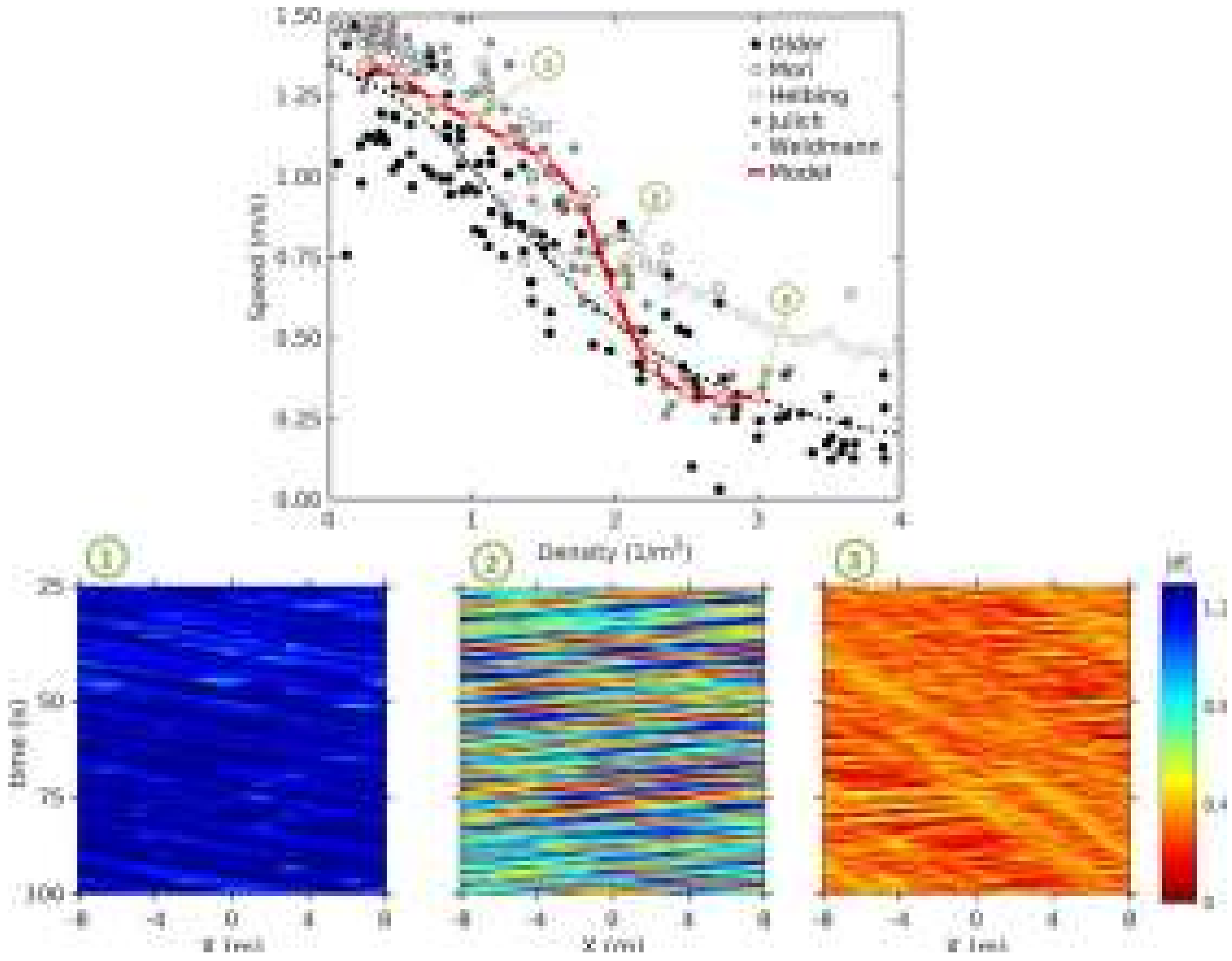
Effect of courtesy



Our TTC-based model with courtesy

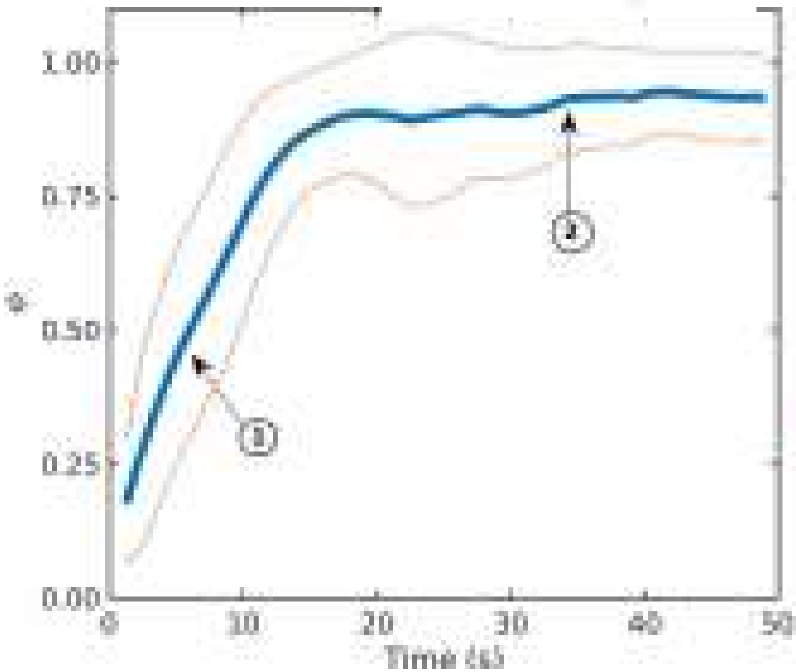
Fundamental diagram

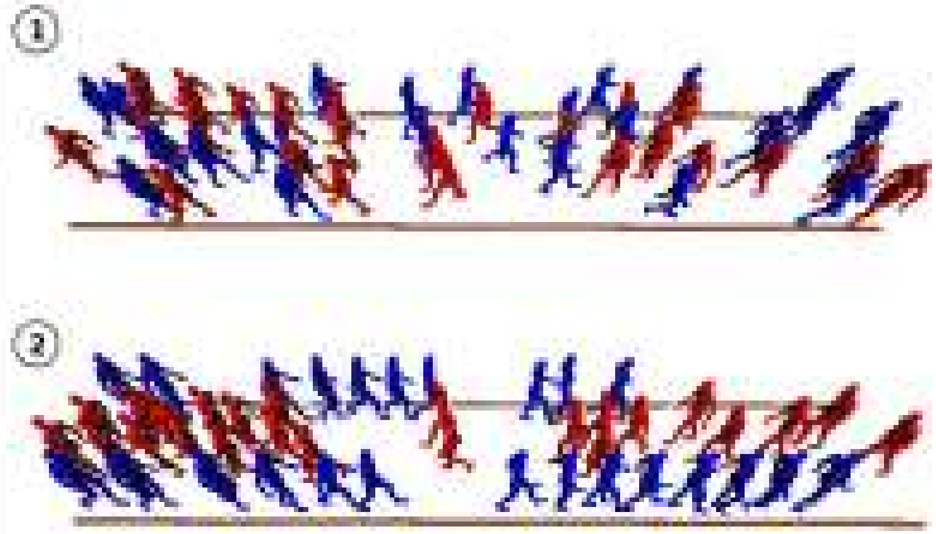
It also captures prominent collective effects



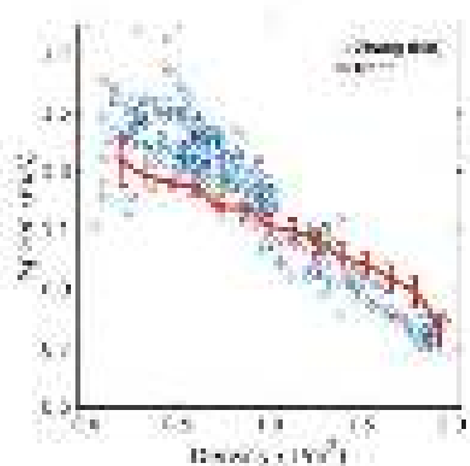
Lane formation

$$\varphi(t) = \frac{1}{N}\sum_{i}^{N} \Delta_i(t) \quad \text{where} \quad \Delta_i(t) = \left(\frac{n_+^i(t) - n_-^i(t)}{n_+^i(t) + n_-^i(t)}\right)^2 \in [0, 1]$$





(vis. : INRIA’s Chaos software, Julien Pettré and colleagues)

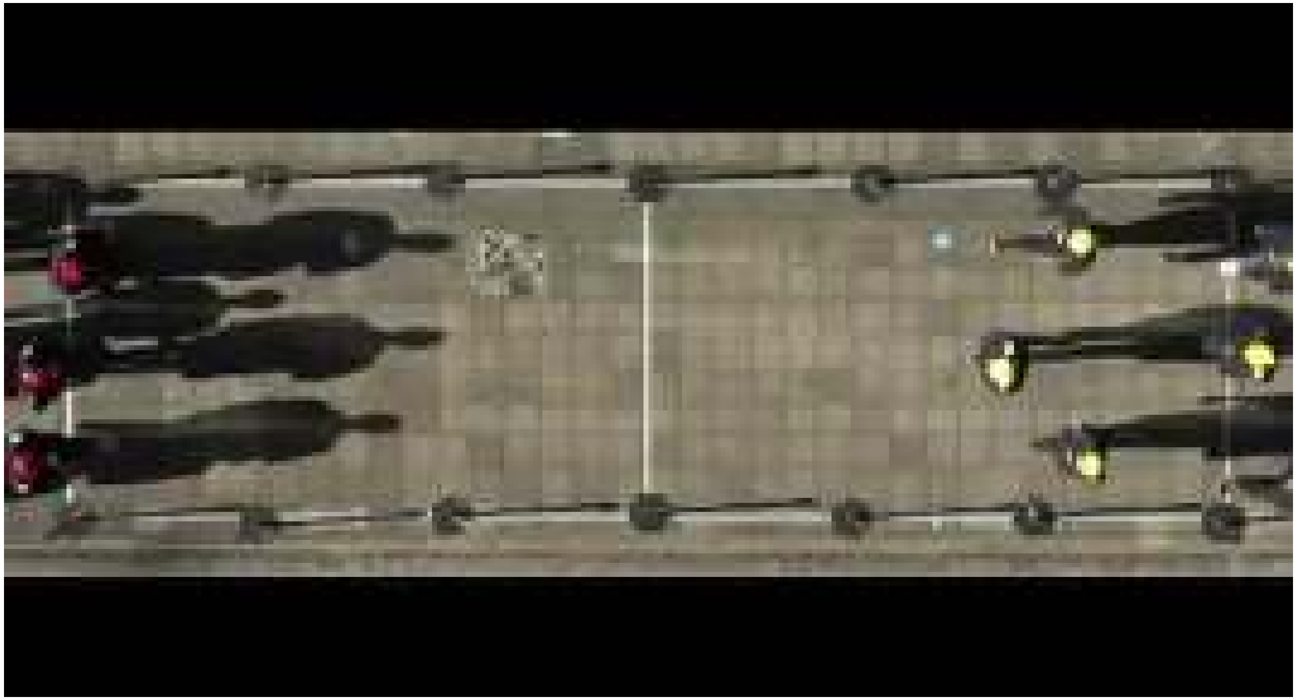


Fundamental diagram for bidirectional flow

Complex geometry

(vis. : INRIA's Chaos software)

Distraction *deteriorates avoidance maneuvers in bidirectional flows*



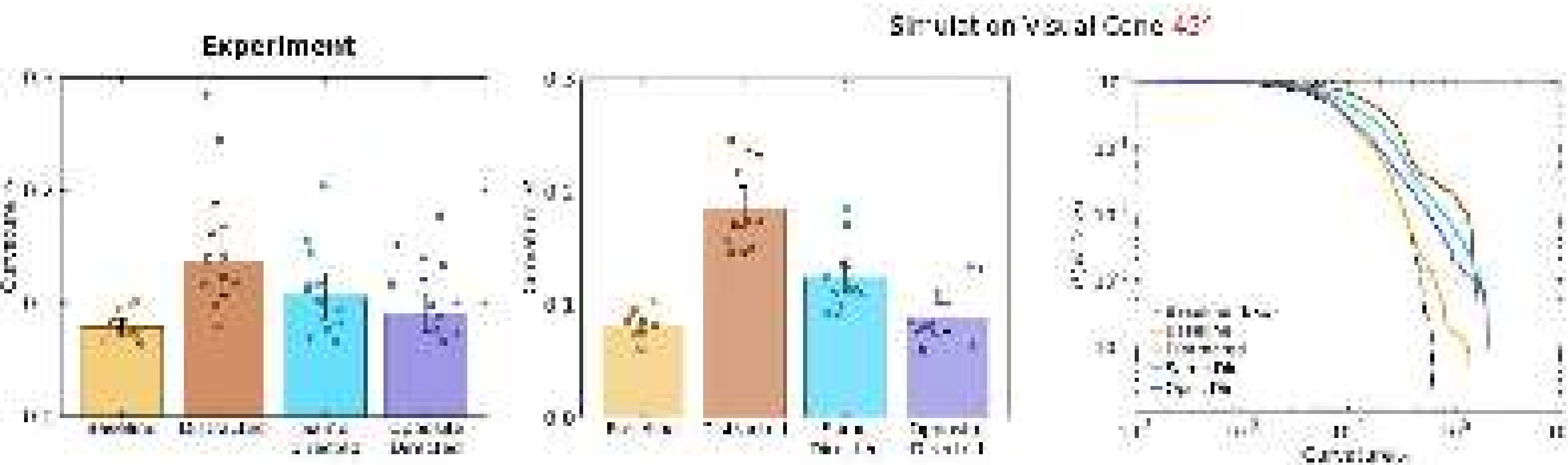
Murakami, Feliciani, Nishiyama, Nishinari, *Science Advances* (2021)

Distraction *deteriorates avoidance maneuvers in bidirectional flows*

Distraction → less frequent updates of the decisional layer (every 0.8s for distracted agents, compared to every 0.1s for standard ones)

Assess how chaotic the flow is using a pseudo-curvature $\left\| \frac{\partial T}{\partial t} \right\| \equiv^{def} \left\| \frac{T_2 - T_1}{dt} \right\|$

The results did not match at first… until a sinusoidal swaying motion (gait) was superimposed on the numerical trajectories



Classification of crowd flows using dimensionless numbers

- Rather extensive validation of the model...
  ... but somewhat too complex to handle theoretically

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⇒ introduce **dimensionless numbers** to gauge which processes really matter

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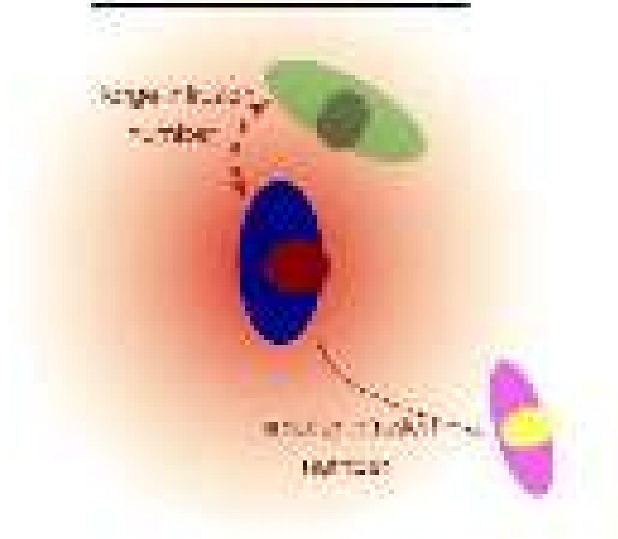
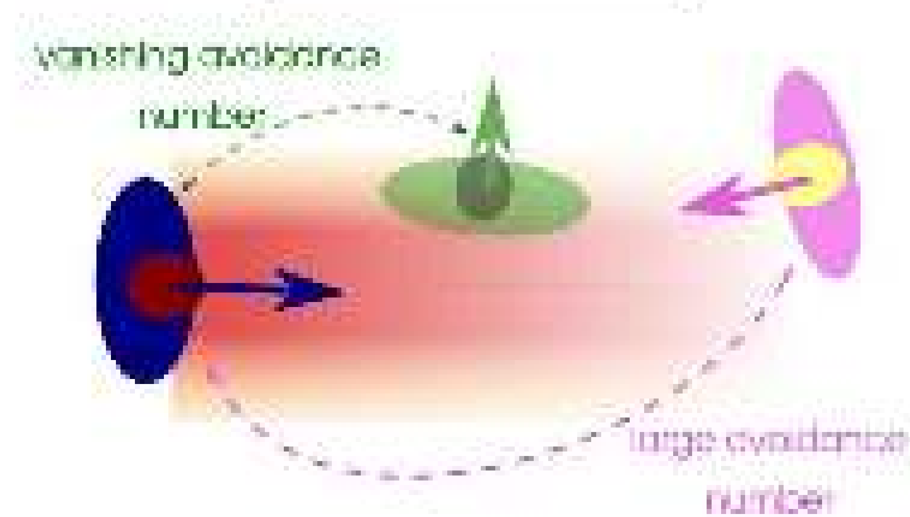
$$In_j = \sum_{<k,j>} \underbrace{\left(\frac{r_{soc} - r_{min}}{r_{jk} - r_{min}}\right)^2}_{In_{jk}}$$

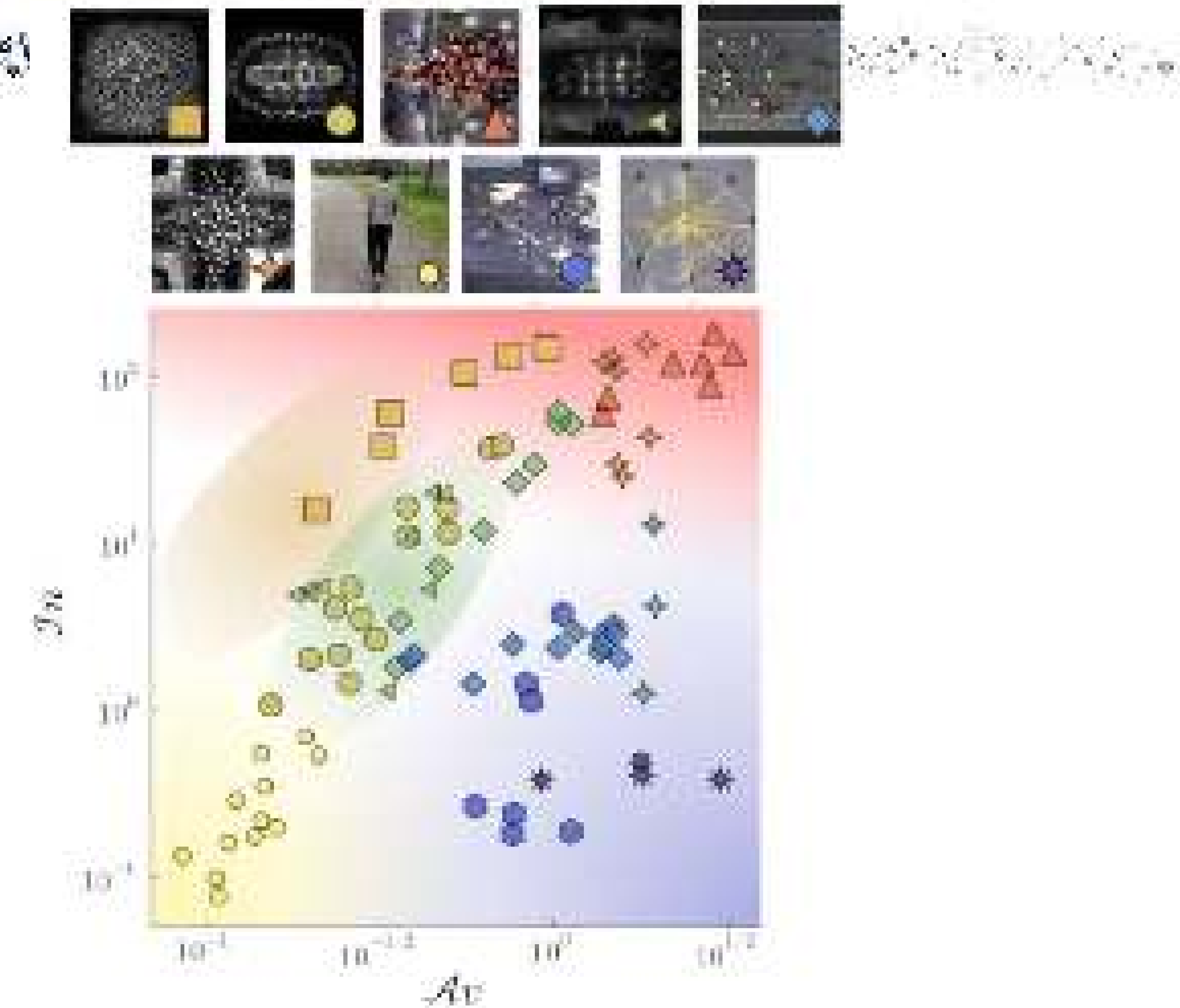
Intrusion number



$$Av_j = \max_k \underbrace{\frac{\tau_0}{\mathrm{TTC}_{jk}}}_{Av_{jk}}$$

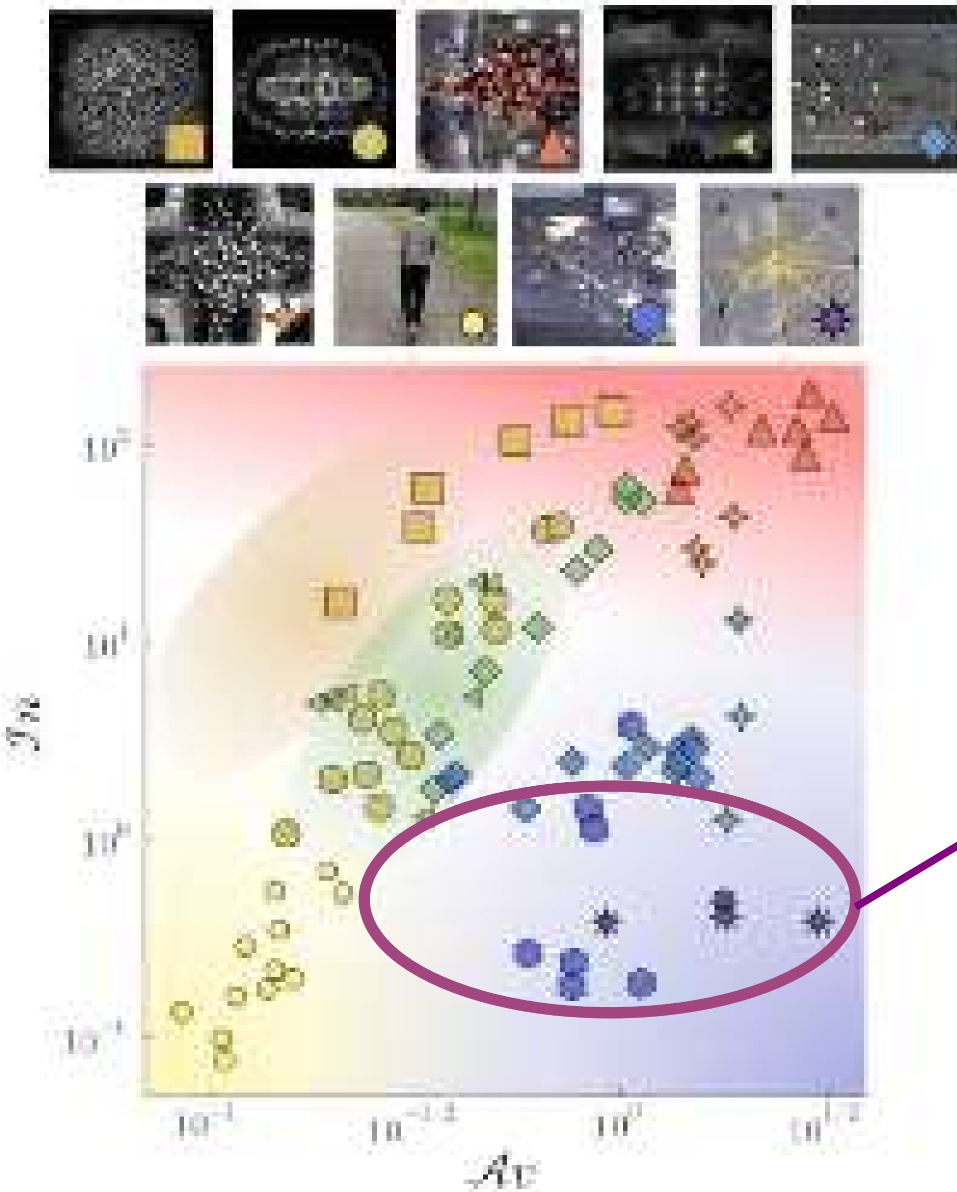
Avoidance number



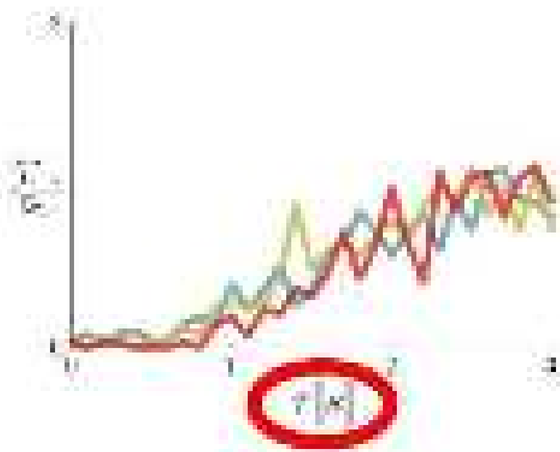


Cordes, Schadschneider, Nicolas, *PNAS Nexus* (2024)

Crowd arrangement

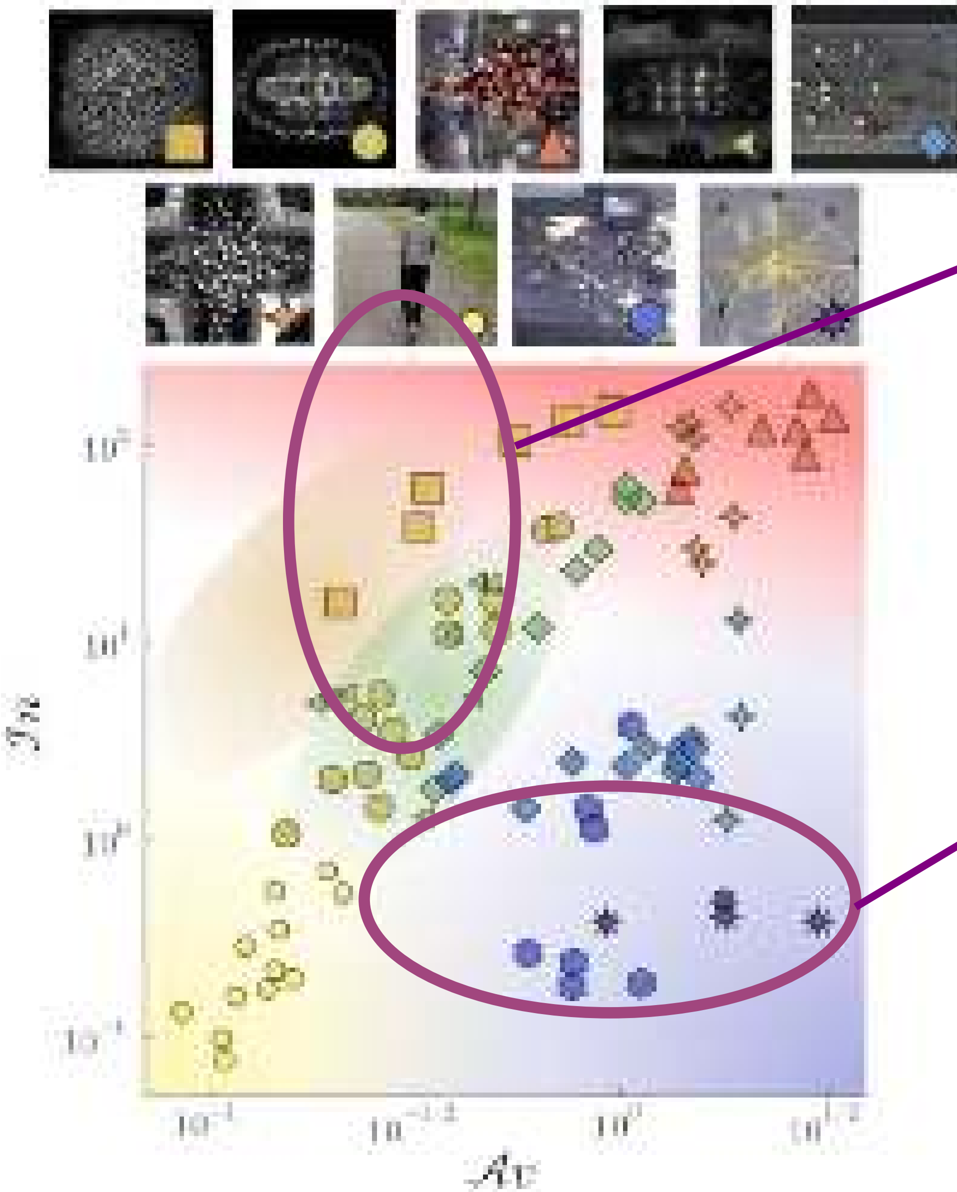


*Pedestrians' spacings are best measured in **TTC space***

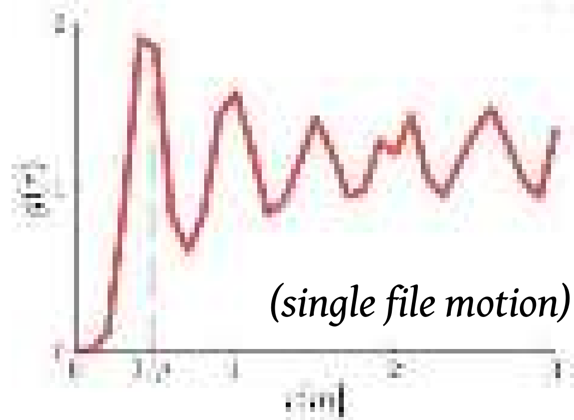


Pair distribution function

Crowd arrangement



*Pedestrians' spacings are best measured by their **Euclidean distances***



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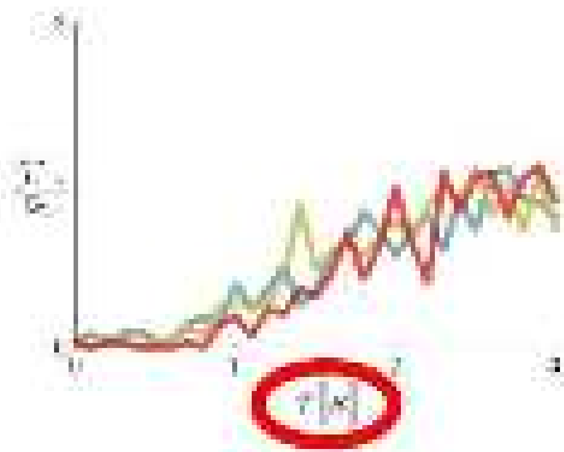


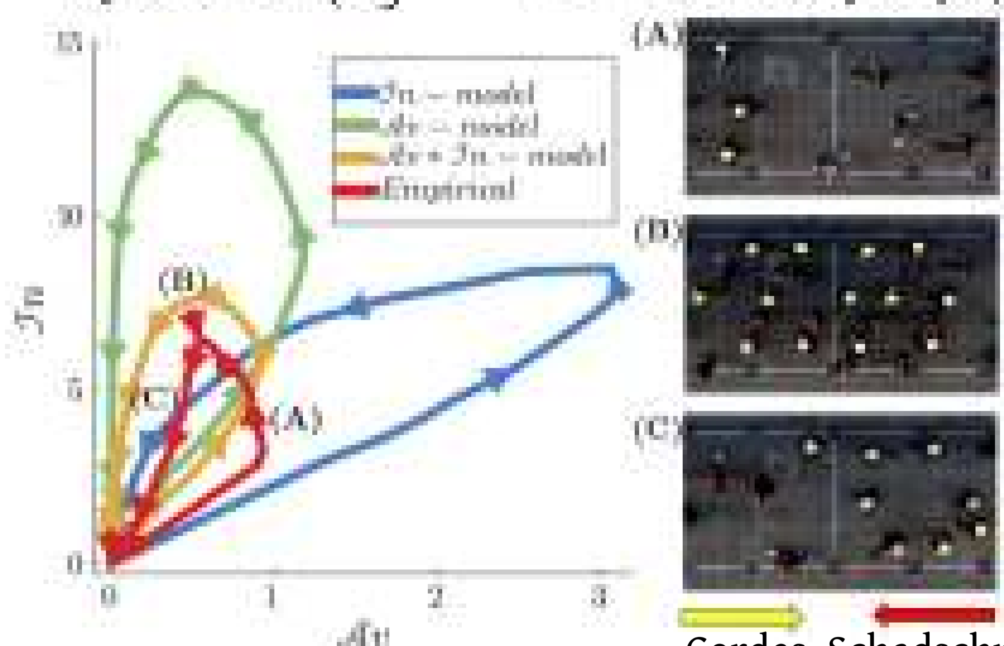
Perturbative expansion *à la Landau*

$$C_j\left[\{\boldsymbol{r}_j\}_{t=0}^{T}\,\middle|\,\{\boldsymbol{r}_{-j}\}\right] \rightsquigarrow C_j\left(\mathcal{I}n_j(\boldsymbol{v}),\mathcal{A}v_j(\boldsymbol{v})\right)$$

where $\mathcal{I}n_j(\boldsymbol{v})$ is evaluated at prospective position $\boldsymbol{r}_j+\boldsymbol{v}\delta t$
$\mathcal{A}v_j(\boldsymbol{v})$ is evaluated with test velocity $\boldsymbol{v}$

After expanding,

$$C_j(\boldsymbol{v}) \approx \left(\boldsymbol{v}_j^{\text{dès}} - \boldsymbol{v} + \beta\nabla\mathcal{I}n_j(\boldsymbol{r}_j)\right)^2 + \alpha\mathcal{A}v_j(\boldsymbol{v})$$



Cordes, Schadschneider, Nicolas, *PNAS Nexus* (2024)

Conclusion

- Statistical physical grasp of active matter has made tremendous progress...

  ... but actual interactions between entities may be more complex

- Decisions can be captured by game theory or using times to collision as a proxy

  ⇒ An emerging avenue of physics, in the continuation of active matter but with fundamental differences that are still to be fully grasped

Acknowledgments

- **Iñaki ECHEVERRÍA-HUARTE** (Univ. Navarra)
- **Cécile APPERT-ROLLAND, Thibault BONNEMAIN, Matteo BUTANO, Antoine SÉGUIN, Denis ULLMO** (Univ. Paris-Saclay)
- **Marcelo KUPERMAN & Sebastian BOUZAT** (Bariloche)
- **Oscar DUFOUR, David RODNEY, Alexis RAULIN FOISSAC** (Univ. Lyon 1)
- **Jakob CORDES, Mohcine CHRAIBI** (Jülich) ; **Benoit GAUDOU & Nicolas VERSTAEVEL** (Toulouse) ; **Antoine TORDEUX** (Wuppertal),

Abstract submission deadline : **May 31st**

https://tgf2024.sciencesconf.org/